Laia Saló i Nevado

PROBLEM SOLVING AND USE OF MATHEMATICS AT WORK

Hanging around farmers and cabinetmakers

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Problem Solving and Use of Mathematics at Work
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Abstract
The purpose of this dissertation is to examine problem solving and mathematical knowledge used at work. Two different work settings were explored: a farm and cabinetmaker workshops. The research focused on the participants' perspectives regarding the problem-solving situations they face and the mathematical knowledge they use at work.

The main research questions behind this dissertation were: what are the mathematics that farmers and cabinetmakers use at work? What are the problem-solving situations that farmers and cabinetmakers encounter at work? How do problem-solving situations influence other ongoing processes?

This dissertation consists of three studies that were developed based on the following objectives. The objective for the first study was to investigate a natural setting where the use of mathematics was not obvious, emphasizing the process that farmers were involved in by analyzing different situations. The objective for the second study, in a setting where mathematics was more obvious, i.e., the workshop of a cabinetmaker, was to find out what kind of mathematics was used in cabinetmakers' everyday work and how problem solving and finding solutions to emergent problems were intertwined in the work. The objective for the third study was to shed some light on what role problem-solving processes played in creative and design woodworking processes.

The theoretical background of this dissertation draws from vernacular mathematics and problem solving. At work, mathematical knowledge is used for both routine tasks and problem-solving situations, and thus problem solving appears to be a significant component of numeracy. The concept of problem solving became central to my work. Earlier studies on the process of problem solving indicated similarities with other processes as the creative process and a connection to the design of a product.

An ethnographic approach was used and the main methods for collecting the data were participant observation, shadowing, and interviews. The participants were two farmers and four cabinetmakers in three separate studies. The inductive analysis of the data from the study with the farmers showed that most of the mathematical knowledge in use and the farmers' behavior focused on quantity, structure, space, and transformations along with problem solving. The data indicated that farmers faced problem-solving situations in dealing with distribution of spaces or in finding suitable objects as measuring tools for feeding the animals. The farmers' reasoning and solutions were based on their own experience within the specific context of the farm. Similarly, the mathematics that cabinetmakers identified and used in their work were in most cases very basic, and therefore this finding was in line with previous studies about mathematics at work. However, the data revealed as well that most ill-structured problem-solving situations appeared when the cabinetmakers had to create a jig. Jigs were context-situated, open-ended and intertwined with a mathematical knowledge component. The analysis indicated that the process of problem solving and the creative process shared similarities, but they were not identical. The data supported the idea that novelty is a condition for making the creative process possible, the same way that viability of the solution is a condition for problem solving. Problem solving influences the length, precision and progress of the design process. This study offers a new reflection on the role of problem solving as a mediating process between creative and design processes.

Keywords: problem solving, mathematical knowledge in use, numeracy, creative process, design process, jig

Helsingin yliopisto, Kasvatustieteellinen tiedekunta
Kasvatustieteellisiä tutkimuksia 107

Laia Saló i Nevado

Ongelmanratkaisu ja matematiikan käyttö työssä
Maanviljelijöiden ja puuseppien työtä tarkkailemassa

Tiivistelmä

Tässä väitöskirjassa tutkitaan työssä käytettyä ongelmanratkaisua ja matemaattista tietoa. Tutkimuksessa seurataan maanviljelöiden työtä tiloillaan ja puuseppien työtä pajoissaan. Tutkimus keskittyy osallistujien näkemyksiin työssään kohtaamista ongelmanratkaisutilanteista sekä matemaattisesta tiedosta, jota he työssään käyttävät.

Väitöstutkimuksen pääkysymykset ovat seuraavat: Millaista matematiikka maanviljelijät ja puusepät käyttävät työssään? Millaisia ongelmanratkaisutilanteita maanviljelijät ja puusepät kohtaavat työssään? Miten ongelmanratkaisutilanteet vaikuttavat muihin työhön liittyviin prosesseihin?

Tutkimus on kolmeosainen. Ensimmäisessä osassa tutkitaan sellaisia luonnollisia tilanteita, joissa matematiikan käyttö ei ole ilmeistä. Näiden tilanteiden analyysi painottuu prosesseihin, joita maanviljelijät käyvät tilanteissa läpi. Tutkimuksen toisessa osassa selvitetään, millaista matematiikkaa puusepät käyttävät ammatissaan ja millainen rooli työssä on ongelmanratkaisulla. Tutkimushavainnot on tehty sellaisissa tilanteissa, joissa matematiikan käyttö on ilmiselvää, kuten puusepän pajatyöskentelyssä. Kolmas tutkimus valaisee ongelmanratkaisuprosessin merkitystä luovassa suunnittelutyössä.

Tutkimuksen teoreettinen tausta on arjen matematiikassa ja ongelmanratkaisussa. Työssä matemaattista tietoa käytetään sekä rutiinitehtävissä että ongelmanratkaisussa, ja siksi ongelmanratkaisu vaikuttaa olevan merkittävä osa laskutaitoa. Ongelmanratkaisusta muodostuikin työni ydin. Aiemmat ongelmanratkaisua käsittelevät tutkimukset osoittavat, että ongelmanratkaisuprosessissa on sekä samankaltaisuuksia luovan prosessin kanssa että yhteyksiä tuotesuunnitteluun.

Käytin työssäni etnografista lähestymistapaa ja keräsin tietoa tarkkailemalla, seuraamalla ja haastattelemalla kohteitani. Kolmen eri tutkimuksen kohteina oli kaksi maanviljelijää ja neljä puuseppää. Maanviljelijöitä tarkkailemalla keräämieni tietojen induktiivinen analyysi osoitti, että valtaosa maanviljelijöiden matemaattisesta tiedosta ja toiminnasta keskittyi ongelmanratkaisun ohella määrään, rakenteeseen, tilaan ja muunnoksiin. Maanviljelijät kohtasivat ongelmanratkaisutilanteita järjestellessään tilankäyttöä tai valitessaan eläinten ruokkimiseen sopivia mittausvälineitä. Maanviljelijöiden päättely ja ratkaisut perustuivat heidän omiin kokemuksiinsa nimenomaan maatilan kontekstissa.

Myös puuseppien työssään käyttämä matematiikka oli hyvin perustasoista, ja niinpä tutkimuksen tulos onkin hyvin samansuuntainen aiempien matematiikkaa työssä käsittelevien tutkimusten tulosten kanssa. Tutkimus kuitenkin osoittaa, että kaikkein monimutkaisimmat, huonosti jäsentyneet ongelmanratkaisutilanteet tulivat vastaan silloin, kun puuseppien oli valmistettava jigi eli jotakin tiettyä työvaihetta varten rakennettu työkalu tai-väline. Jigit olivat tilannesidonneisia ja ratkaisuiltaan avoimia, ja niihin liittyi matemaattisen tiedon eri osa-alueita.

Analyysi osoittaa, että vaikka ongelmanratkaisuprosessissa ja luovassa prosessissa on yhtäläisyyksiä, ne eivät ole identtisiä. Tulokset tukivat oletusta, että luovan prosessin ehtona on tilanteen uutuus ja ensikertaisuus, samalla tavalla kuin toteuttamiskelpoisuus on ongelman ratkaisun ehto. Ongelmanratkaisuun vaikuttavat suunnitteluprosessiin käytetty aika, tarkkuus ja edistyminen. Tämä tutkimus tuo esiin uuden tavan tarkkailla ongelmanratkaisun merkitystä luovan prosessin ja suunnittelun välisenä toimintana.

Avainsanat: ongelmanratkaisu, matemaattinen tieto käytössä, laskutaito, luova prosessi, suunnitteluprosessi, jigi

Universitat de Hèlsinki, Facultat de Ciències de l'Educació
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Laia Saló i Nevado

La resolució de problemes i l'ús de les matemàtiques al lloc de treball
Entre grangers i fusters

Resum

L'objectiu d'aquesta tesi és examinar la resolució de problemes i els coneixements matemàtics en dos llocs de treball ben diferents: una granja i diverses fusteries. La investigació se centra en les perspectives dels participants sobre les pròpies situacions de resolució de problemes i els coneixements matemàtics que utilitzen a la feina.

Les preguntes d'investigació de les que parteix aquesta tesi són: Quins coneixements matemàtics fan servir els grangers i els fusters a la feina? Quin tipus de situacions els obliga a resoldre problemes? En quina mesura la resolució de problemes condiciona altres processos de la vida laboral?

Aquesta tesi consta de tres estudis. L'objectiu del primer estudi és investigar un entorn natural on l'ús de les matemàtiques no és obvi, posant èmfasi en el procés que els grangers experimenten mitjançant l'anàlisi de diferents situacions. El segon estudi es desenvolupa en un entorn on les matemàtiques són més evidents: el taller d'un fuster. L'objectiu d'aquest segon estudi és esbrinar quin tipus de matemàtiques s'utilitzen en el treball diari dels fusters i com resolen els problemes emergents. El tercer estudi fa aportacions vinculades al paper que juga la resolució de problemes en els processos de disseny i creació de treballs en fusta.

El marc teòric d'aquest treball prové del coneixement matemàtic vernacle i del camp de la resolució de problemes. En el món laboral les matemàtiques s'usen tant per dur a terme tasques rutinàries com per resoldre situacions que plantegen reptes, per tant, la resolució de problemes esdevé un element significatiu de l'alfabetització digital de l'individu. Així doncs el concepte de resolució de problemes esdevé clau en el meu treball. Altres estudis sobre aquest procés han demostrat que existeixen semblances entre els procediments que s'empren per resoldre problemes i els que s'utilitzen en els processos de creació i disseny d'un producte.

Aquest estudi adopta un enfocament etnogràfic per analitzar les dades que s'han recollit a través d'instruments com ara l'observació participant, l'aprenentatge a través de l'observació, la participació i les entrevistes. El corpus pel conjunt dels tres estudis el composen dos grangers i quatre fusters. Les dades de l'estudi fet amb els grangers s'han analitzat amb mètodes inductius i els resultats obtinguts indiquen que els grangers fan servir coneixements matemàtiques per dur a terme accions lligades als conceptes de quantitat, espai, transformació i també a la resolució de problemes. Per exemple, els grangers han de resoldre problemes

vinculats a la distribució dels espais o a la cerca d'objectes que els puguin servir com a eines per donar menjar als animals. Els seus processos de raonament i les solucions que proposen es basen en les seves pròpies experiències viscudes en l'entorn laboral de la granja. En l'estudi fet amb els fusters s'observa un comportament semblant i les matemàtiques que empren són, en la majoria de casos, molt bàsiques. Aquests resultats coincideixen amb les troballes d'altres estudis sobre l'ús de les matemàtiques al lloc de treball. No obstant, les dades obtingudes són significatives perquè també demostren que quan els fusters tenen la necessitat de crear algun tipus d'eina artesanal ho fan a partir de resoldre problemes que estan mal plantejats, tot i estar contextualitzats, presentar solucions obertes i basar-se en l'aplicació de coneixements matemàtics. L'anàlisi assenyala que els procediments de resolució de problemes són semblants als processos creatius, però no són idèntics. Les dades indiquen que la noció de novetat és una condició intrínseca en un procés creatiu. En canvi, en un procés de resolució de problemes aquest element clau és la viabilitat de la solució. És a dir, la resolució de problemes té un impacte sobre la durada, la precisió i el progrés de tot procés creatiu. En aquest sentit, l'estudi aporta nous elements de reflexió sobre el paper de la resolució de problemes com a procediment de mediació entre els processos de creació i disseny.

Paraules clau: resolució de problemes, ús de coneixements matemàtics, alfabetització numèrica, procés creatiu, procés de disseny, eines de fabricació artesanal/plantilles guia

Universidad de Helsinki, Facultad de Ciencias de la Educación
Estudios en Educación número 107

Laia Saló i Nevado

La resolución de problemas y el uso de las matemáticas en el lugar de trabajo
Entre granjeros y carpinteros

Resumen
Esta tesis estudia la resolución de problemas y el uso del conocimiento matemático en dos lugares de trabajo muy diferentes: una granja y varias carpinterías. La investigación se centra en la percepción que tienen los participantes respecto a la resolución de problemas y el uso de los conocimientos matemáticos en diferentes situaciones laborales. Las principales preguntas de investigación son las siguientes: ¿Qué conocimientos matemáticos usan los granjeros y los carpinteros en el trabajo? ¿Qué tipo de situaciones les obliga a resolver problemas? ¿En qué medida la resolución de problemas condiciona otros procesos de la vida laboral?

Esta tesis es un compendio de tres estudios. El objetivo del primer estudio es investigar un entorno natural en el que el uso de las matemáticas no es obvio: una granja. Este estudio enfatiza el proceso que los granjeros experimentan al desarrollar sus tareas laborales mediante el análisis de diferentes situaciones. El segundo estudio se desarrolla en un entorno donde el uso de las matemáticas parece más justificado: el taller de un carpintero. El objetivo del segundo estudio es averiguar qué tipo de matemáticas emplea el carpintero para llevar a cabo su trabajo diario, y cómo resuelven los problemas que surgen. El tercer estudio hace aportaciones vinculadas al papel que juega la resolución de problemas en los procesos de diseño y creación de trabajos en madera.

El marco teórico de esta tesis proviene del conocimiento matemático vernáculo y del campo de la resolución de problemas. En el mundo laboral, las matemáticas se usan tanto para llevar a cabo tareas rutinarias como para resolver situaciones que plantean retos. Por tanto, la resolución de problemas se convierte en un elemento significativo de la alfabetización digital del individuo. Así pues, el concepto de resolución de problemas es clave en mi trabajo. Otros estudios sobre este proceso han demostrado que existen semejanzas entre los procedimientos que se emplean para resolver problemas y los que se utilizan en los procesos de creación y diseño de un producto.

Este estudio adopta un enfoque etnográfico para analizar los datos que se han recogido a través de instrumentos como la observación participante, el aprendizaje a través de la observación, la participación y las entrevistas. El corpus por el conjunto de los tres estudios lo componen dos granjeros y cuatro carpinteros. Los datos del estudio realizado con los granjeros se han analizado con métodos inductivos y los resultados obtenidos indican que los granjeros usan conocimientos matemáticos para llevar a cabo acciones ligadas a los conceptos de cantidad, espacio, transformación y también a la resolución de problemas. Por ejemplo, los granjeros deben resolver problemas vinculados a la distribución de los espacios o en la búsqueda de objetos que puedan servir como herramientas para dar de comer al ganado. Sus procesos de razonamiento y las soluciones que proponen se basan en experiencias propias vividas en el entorno laboral de la granja. En el estudio realizado con los carpinteros se observa un comportamiento similar y las matemáticas que emplean son, en la mayoría de los casos, muy básicas. Estos resultados coinciden con los hallazgos de otros estudios sobre el uso de las matemáticas en el lugar de trabajo. Sin embargo, los datos obtenidos son significativos porque también demuestran que cuando los carpinteros tienen la necesidad de crear algún tipo de herramienta artesanal lo hacen a partir de resolver problemas que están mal planteados, a pesar de estar contextualizados, presentar soluciones abiertas y basarse en la aplicación de conocimientos matemáticos. El análisis señala que los procedimientos de resolución de problemas son similares a los procesos creativos, pero no son idénticos. Los datos indican que la noción de novedad es una condición intrínseca en un proceso creativo. En cambio, en un proceso de resolución de problemas este elemento clave es la viabilidad de la solución. Es decir, la resolución de problemas tiene un impacto sobre la duración, la precisión y el progreso de todo proceso creativo. En este sentido, el estudio aporta nuevos elementos de reflexión sobre el papel de la resolución de problemas como procedimiento de mediación entre los procesos de creación y diseño.

Palabras clave: resolución de problemas, uso de conocimientos matemáticos, alfabetización numérica, proceso creativo, proceso de diseño, herramientas de fabricación artesanal/plantilla guía

Acknowledgements

If you are reading this, it means that it is over, done. Problem solved. The path of this doctoral thesis has been long, personal and arduous but, at the same time, captivating and enriching. I have relished many moments. Yet, it has taken turns and re-directions several times. It has been hard and tough not to give up as it has mostly been done in addition to my everyday work, during the evenings and weekends. At times it has been demanding and challenging especially when my mind got jammed, blocked without a way see how to get it untangled.

This story of *Problem Solving and Use of Mathematics at Work* has different actors and characters playing different roles, coming and going as in real life. First and foremost, I want to thank the farmers and cabinetmakers who agreed to participate in my studies. They shared, showed, and provided me with the knowledge about their life and work that I could have never grasped alone.

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In the last stage of this lengthy path, I am grateful to the pre-examiners of this thesis, Professor Emeritus Diana Coben from King's College London and Professor Javier Díez-Palomar from Universitat de Barcelona for their detailed and constructive reports and feedback. I am equally indebted to Professor Peter Liljedahl from Simon Fraiser University for accepting to be the opponent in the defense and for being a source of inspiration.

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Donec Perficiam.

In Vantaa, 19 March 2021
On my dad Agustí's 70th birthday

Laia Saló i Nevado

LIST OF ORIGINAL PUBLICATIONS

This thesis is based on the following publications:

I Saló i Nevado, L., Holm, G., & Pehkonen, L. (2011). Farmers do use mathematics: The case of animal feeding. *Nordic Studies in Mathematics Education, 16*(3), 43–63.

II Saló i Nevado, L., & Pehkonen, L. (2018). Cabinetmakers' Workplace Mathematics and Problem Solving. *Vocations and Learning, 11*(3), 475-496. https://doi.org/10.1007/s12186-018-9200-8

III Saló i Nevado, L., Pehkonen, L. & Salminen, M. (2020). Workplace Problem Solving within the Design Process, *Techne Series – Research in Sloyd Education and Craft Science A, 27*(1), 36-51.

The publications are referred to in the text by their roman numerals.

Author's contribution

Laia Saló i Nevado is the lead author in publications I, II and III. She was the main contributor to all aspects (i.e., research idea and conception, questions, theoretical approach, data collection, writing and interpreting the findings, finalization, and revision) of these papers. Laia Saló i Nevado is also solely responsible for writing the summary chapter of this thesis.

Abbreviations

cf.	conferatur (compare)
e.g.,	exempli gratia (for example)
et al.	et alii (and others)
etc.	et cetera (and so forth)
i.e.,	id est (that is)

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1 INTRODUCTION

"For the things of this world cannot be made known
without a knowledge of mathematics"
Opus Majus, 1267
Roger Bacon (1214-1292)

"Each problem that I solved became a rule,
which served afterwards to solve other problems"
Discours de la Méthode, 1637
René Descartes (1596-1650)

Mathematical knowledge is included in our everyday settings (Nunes, 1992; Carraher & Schliemann, 2002; Wedege, 2010). Mathematical knowledge can be conceived as the observation of the particular rules, use of concepts and tools. It connects to values and embraces the knowledge and behavior that deal with change, structure, space and transformation (Van Oers, 2001). This characterization of the term brings the understanding that mathematical knowledge is not a mere abstract concept, but it also surges from reality, as a code to explain and justify reality.

FitzSimons (2008) relates mathematical knowledge to numeracy, which at first hand may seem obvious, but it is not a simple position. Numeracy stands as the mathematical knowledge in practice, as the ideas and the techniques in the different situations of everyday human life. FitzSimons argues that a person's numeracy is based on common sense, and is context-specific and dependent. She also indicates that it is aimed at concrete, immediate and relevant goals, yet it is molded and shaped by the change and development inherent to everyday life.

Everyday life for an adult revolves around work and leisure time. Working life and know-how at work are in constant change, which is partially triggered by the technological advances (Carnevale & Desrochers, 2003; Hoyles et al., 2010; Tall, 2013). These changes, along with the efficacy and resilience requirements at work, lead to questions about what competences are needed for people to succeed in working life and what should be taught at school. The question remains as to whether the focus should be on context-bound knowledge of a specific job or elsewhere (FitzSimons, 2014). It would make sense to focus on the contextual knowledge to a certain extent, but if development and change are considered, it is not the most favorable option. FitzSimons (2014) reminds us that one of the most important factors at work is the continuous need to learn and solve problems for which one has no previous experience nor education. To solve such problems, professionals must produce and use new kinds of knowledge and skills. In addition, they need to find creative and innovative solutions in which mathematical knowledge often plays a significant role. In line with these arguments, there are studies suggesting that the criteria for employment have changed over the last few decades. Among other skills, employers look for reliability, the ability to access and handle information, cooperation, communication and, again, problem-solving skills in their workers. Some studies claim that problem-solving skills are one of the core competencies in the 21st century workplace (Belland, 2013; Csapó & Funke, 2017). FitzSimons argues that the main skills needed at work are critical thinking, complex problem solving and creativity (2019). This recurrence of problem solving sparked my curiosity towards research on what problem-solving situations appear in workplaces and what kind of mathematical knowledge is used. The workplaces are the contexts for the use of mathematical knowledge; in other words, numeracy in practice. At the same time, this numeracy is considered a form of critical thinking (Jain & Rogers, 2019). Furthermore, the vernacular mathematics of a labor group is the mathematical knowledge used by a specific labor group at work (Eglash, 1997), which, in other words, is their numeracy skill to deal with problem solving.

This dissertation examines problem solving and the mathematical knowledge used at work in two different labor group cases (i.e., vernacular mathematics of farmers and cabinetmakers) through an ethnographic approach. The research aims to find out what mathematical knowledge farmers and cabinetmakers use at work and what problem-solving situations these two parties encounter in their daily work. In addition, the research examines how problem-solving situations influence other ongoing processes.

This dissertation consists of a summary and three published studies. The summary begins by presenting the theoretical framework of my study from the concept of mathematical knowledge in use and numeracy at work (chapter 2). This leads to an examination of problem solving as a central concept, and its similarities with the creative process as well as the design process. Next the research task is described, including the research questions, aims and objectives (chapter 3). An overview of each of the studies is presented, followed by the details of the methodology used (chapters 4 and 5). Lastly, the findings are presented along with a discussion and concluding notes (chapters 6, 7 and 8).

2 THEORETICAL FRAMEWORK

2.1 Mathematical Knowledge and Vernacular Mathematics

Different societies adapt to their different realities and contexts. With the analysis of reality in the attempt to adapt, humans have been able to generate and conceive a collection of rules, patterns, parameters and standards that respond to understanding what is going on in the reality that they live. This group of rules, patterns, parameters and standards help in the development and adaptation of society to everyday life. This is how mathematics can be identified, and why different societies may use different mathematical knowledge (e.g., Arabic, Chinese, Babylonian, Roman or Mayan). Thus, one can understand that there are not "many mathematics", but different uses of mathematics and different types of mathematical knowledge emerging from different needs (see Van Oers, 2001; D'Ambrosio, 2001; cf. Greiffenhagen & Sharrock, 2008; Tall, 2013, chapter 9). Accordingly, mathematics can be regarded as a cultural construct (see Iseke-Barnes, 2000; Van Oers, 2001).

In the present, one can consider that the mathematical knowledge transferred or taught in a formal setting (schools) is selected and responds to what is considered the "basic packet", which is necessary for the functional numeracy of a citizen (FitzSimons, 2013; 2013a). D'Ambrosio suggests similarly that mathematics enables one's full realization as a citizen (D'Ambrosio, 2001, p. 68). However, this mathematical knowledge is not static and constant but has been adapted to suit different needs. It is modified by factors such as technological advances, new devices and new knowledge resulting from the development and advancement of the old (Tall, 2013): e.g., in ancient times, basic calculations of large numbers were performed with an abacus, and nowadays, having gone through the paper-pencil stage, computers and calculators are used for the same purpose. One can claim that the need for and use of large number calculations has changed over time, but strategies and development have impacted that change. Therefore, the use of mathematical knowledge evolves to serve different practical approaches.

The concept of Ethnomathematics, coined by Ubiratan D'Ambrosio (see D'Ambrosio, 1985; 2001; Rosa & Orey, 2011) includes the prefix ethno-, which denotes cultural, natural and environmental plurality, comprising different forms of knowledge, in this case mathematical knowledge. My understanding is that Ethnomathematics is a concept of vast reach, as it includes all the different practices of all the various cultural groups. It also embraces natural and situational plurality. Thus, with Skovsmose (2015, p. 21), whether Ethnomathematics is exchangeable for the mathematics used by a particular group or not is a question of semantics. In this thesis I wish to steer clear of such controversy, but I also find it necessary to clarify my understanding of the term. I concur with the division of the anthropology of mathematics presented by Eglash (1997). Eglash distinguishes five categories: non-western mathematics, mathematical anthropology, sociology of mathematics, vernacular mathematics and Ethnomathematics. According to Eglash, Ethnomathematics is directly associated with small-scale societies, which leaves those groups outside mathematical professionalism and those communities or individuals not considered as an anthropological group as belonging to the category of vernacular mathematics (1997, p. 81). Thus, this thesis' core is vernacular mathematics, understood as the use of mathematics that a concrete labor group manages at work. In addition, Skovsmose's concluding remark on Ethnomathematics being a research program adjusts my understanding of what the term represents: a different way to examine the social aspects of mathematical knowledge and its cultural dimensions (2015). Yet, the unavoidable connotation that the prefix "ethno" implies race and culture encouraged me to discard the use of the term to designate the core field, which this thesis is concerned with.

2.2 Numeracy

Back in the 1980s, the Cockcroft Report considered the mathematical needs of adults in everyday life and defined numeracy as the ability to manage those needs successfully (Cockcroft, 1982). However, as Evans argues (2000), it is not enough to define numeracy in such a manner. The Cockcroft Report can be considered a turning point, since, as Evans highlights in his study, it uses the term numeracy to refer to mathematics or mathematical knowledge in practical everyday situations. According to Evans, the term numeracy entails the transfer of skills beyond pure mathematical knowledge (2000, p. 12) and makes room for further study of the concept of numeracy (see also Jonas, 2018; PIAAC Numeracy Expert Group, 2009; Tout et al., 2017). In this study, human numeracy is understood as the capacity of a person to use mathematical skills to perform and overcome challenges within society. The mathematical knowledge in use is a part of that person's language and way of thinking. Numeracy includes handling and using numerical information, as well as different techniques to adapt and manage information within a given context. In other words, numeracy is the capacity, ability and competence to use numbers and other mathematical knowledge to solve problems, make decisions and perform in everyday life (Evans, 2000). Consequently, in this study I have used both terms differentiating their meaning since numeracy is not mathematics (mathematical knowledge) and vice versa (Liljedahl & Liu, 2013). They are connected but not interdependent, since mathematics does not require numeracy. Mathematics, as a discipline and a body of knowledge, is partly a requisite for numeracy to be effective. In other words, for numeracy to be functional it is necessary that certain mathematical knowledge should be present. Several other elements are also required, such as the ability to use that knowledge, the ability to interpret and communicate, the ability to deal with information of a varied nature or the capacity to use different methods and tools within a certain context (see also Straesser, 2015).

The Department of Education of the Government of Alberta (Alberta Education, 2019) illustrated the different elements and components of numeracy in a circular figure (see Appendix A: Numeracy Progressions) and included almost everything I consider essential to the concept of numeracy. However, the figure was oriented towards education and learners. The conception that this thesis operates on demands certain modifications to the elements and their position since I aim to foster a broader understanding of numeracy. To this end, inspired by the illustration offered by Alberta Education (2019), I propose the modification in Figure 1, which provides a more general view of the concept of numeracy within everyday life and for the purpose of this study.

Both the Alberta Education illustration (see Appendix A) and Figure 1 present a first conceptual division of numeracy as knowledge and understanding on one side and awareness on the other. For me, numeracy knowledge does not only signify the mathematical knowledge possessed by an individual. I include information, facts, formulae and data as mathematical knowledge. The combination of mathematical knowledge and understanding includes the information, the skills, competences and abilities of an individual. Each of these elements have different levels and are personal to each individual since they depend on factors such as level of education, life experiences or age. There are two main sources of mathematical knowledge, the quantitative and the spatial (see Figure 1). Quantitative information or knowledge is understood as basic and advanced calculations, number sense and use, data collection, patterns and relationships, data systematization, magnitudes, estimation, approximation and probability. Spatial knowledge is based on spatial visualization, measurement, conversion of units, time, location and direction. Other skills, competences and abilities are necessary to put the mathematical knowledge to use. Those encompass the use of strategies, tools and methods and problem solving (essential for an individual to be able to interpret, represent and communicate). The awareness side of the wheel shows that the functionality of numeracy relies on three key elements: the user, who has a personal insight based on their past experiences; the task and its analysis; and the purpose of the situation. All of these are entirely dependent on the context. Both problem solving and context are essential additions to Figure 1 and will gain significance throughout this thesis.

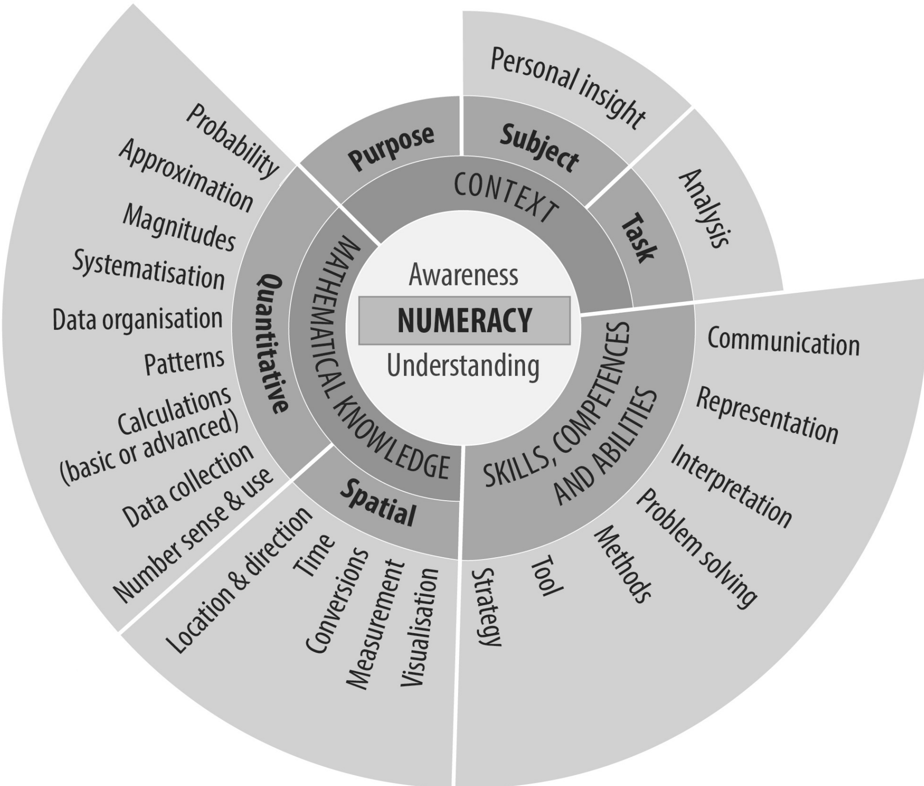


Figure 1. Numeracy within everyday life, modification to Numeracy Progressions (Alberta Education, 2019).

Before any further considerations, it is necessary to make note regarding the use of terms in this thesis. I do not work with the concept of *workplace mathematics*, but instead with *use of mathematics at work* and *mathematical knowledge in use*. Therefore, when I use the term *workplace mathematics,* I am referring strictly to the *use of mathematics in a workplace*. The research is performed in various situations and settings at different workplaces where mathematical knowledge is *used* in one way or another; however, I by no means intend to imply the existence of another kind of mathematics at work. Over the years, and within research on out-of-school mathematics and so-called *everyday mathematics*, some criticism has been directed to the confusion that the use of certain terms might have generated. Greiffenhagen & Sharrock (2008), using mostly Lave's foremost studies *Cognition in Practice* (1988, cited in Greiffenhagen & Sharrock, 2008) among others (see Greiffenhagen & Sharrock, 2008), objected that the significance of research on *everyday mathematics* is limited. They also claim that there are many studies that unknowingly use the term to refer not to new computational and algorithmic findings but to different aspects related to the settings or the context where mathematics is in use. Greiffenhagen and Sharrock assert that the main purpose of everyday mathematics research seems to be aimed at providing contrast to formal mathematics teaching. They acknowledge that it is a difficult achievement since the objectives and goals in those situations are different. Greiffenhagen and Sharrock express their surprise in terms of how little some everyday mathematics studies deal with the aims of school mathematics, when the main purpose of school mathematics would apparently be to teach skills used in everyday life. Then, when the school mathematics algorithms and formulae do not appear in real life situations, it is ironically considered a failure of school mathematics. Greiffenhagen and Sharrock accuse others of following Lave's affirmations (see Taylor, 1991; Masingila, 1994, both cited in Greiffenhagen & Sharrock, 2008, p. 13). I agree with Greiffenhagen and Sharrock that the claims of some studies that their subjects have reinvented or discovered new mathematics are in most cases ambiguous and misleading. Often the case is that the mathematical knowledge is used differently. Thus, my clarification on the use of terms in this thesis was necessary.

2.3 Use of Mathematics at Work

Adults spend a great number of hours at work weekly. Thus, one of the main numeracy contexts within everyday life is the workplace, the location where someone works. In this thesis, I understand work as the tasks allotted at the workplace (i.e., practices, routines and problem-solving situations). Each workplace is unique and the practices that take place there are highly situated (Plowman, Rogers & Ramage, 1995). I consider that the concept of workplace cannot be used generically, since it is bound by the individuals working and the given conditions (e.g., time of the year, location, tools, needs, materials, market demands, etc.). Nowadays, the situation regarding workplaces is more complicated since, in many professions, individuals are not compelled to work in a particular location. It is possible to work in places such as the train, on a bench outdoors, in a cafeteria, at home, in an office, etc. At the same time, there are many workplaces where a specific location is unavoidable, concrete and definite due to the needs of the job, tools, materials, resources or the characteristics of the setting (e.g., a hospital, an artisan's workshop, a bakery, a restaurant, etc.). For this thesis, I selected two professions that require a concrete workplace to execute their work: farmers at their farm and cabinetmakers at their workshop. I will give further details on these workplaces below.

Keogh, Maguire and O'Donoghue (2014; 2018) present a detailed analysis of the factors that contextualize each workplace. These factors are uncovered with ease depending on the level of complexity of different situations (i.e., routine tasks, problem-solving situations) and, at the same time, the mathematical knowledge, skills and competences become, accordingly, either more or less visible. The researchers examine the factors grouped in five dimensions, namely accountability, clarity, familiarity, stressors, and volatility. Keogh et al. describe under accountability factors such as the impact of error, also called tolerance (Wedege, 2002), highly dependent on the tasks and their purpose; the clearance of individuals to make decisions; judgements linked to common sense and experience; the capacity to plan to obtain an optimal solution based on the given demands, conditions and requirements; and responsibility along with liability. Under clarity, related to the goals and aims of the situation, they introduce the factor of vision, according to how meaningful a task is for the worker. It is stated that the precision of the information directly influences the level of creativity developed either in the task development or in the product. Familiarity is influenced by specificity as the fact that various working elements are handled at the same time. In addition, people working together can influence the completion and development of tasks as well as the working roles. Under stressors, Keogh et al. present the constraints at different levels, such as time, materials or tools. In addition, they mention the problem potential, meaning the complexity at work conditioned by the number of problem-solving situations arising. Last but not least, Keogh et al. gather under volatility aspects such as the conditions for change and modification, demand, diversity of tasks, how predictable they are and the risks related to decision-making and the creativity of the outcome (Keogh et al., 2014, pp. 86-94; 2018).

The context, traits and features of the workplace are continuously evolving, and the advance of technology is a highly influential element (Carnevale & Desrochers, 2003; Hoyles et al., 2010). At work, Wedege (2007) argues that, within the technological world, mathematical skills and competences are often only visible when technology changes or is modified; this can be translated into a rupture of the routine and, possibly, can be when a problem emerges (see also FitzSimons, 2013). Wedege suggests that the workers might sometimes be conscious of their numeracy only under such circumstances. However, numeracy is not just the capacity and ability to manage only when dealing with a problem, but a greater embodiment of all those given elements (see Figure 1) in a harmony of actions at different times of the day. Wedege also describes three different intertwined components related to technology (as cited in 2000: 1995 a&b; 2002). The first is technique in terms of equipment, machines and tools, also including cultural technique such as language and time. Second, the work organization, which includes the different tasks, responsibilities and competencies that are given by the structure at a workplace. The third element is the human qualifications, which she describes as the skills and abilities that are required for the technique and the work organization. Within this last component, Wedege includes knowledge of the profession, literacy and numeracy, as well as attitudes such as flexibility and cooperation (2000, p. 128).

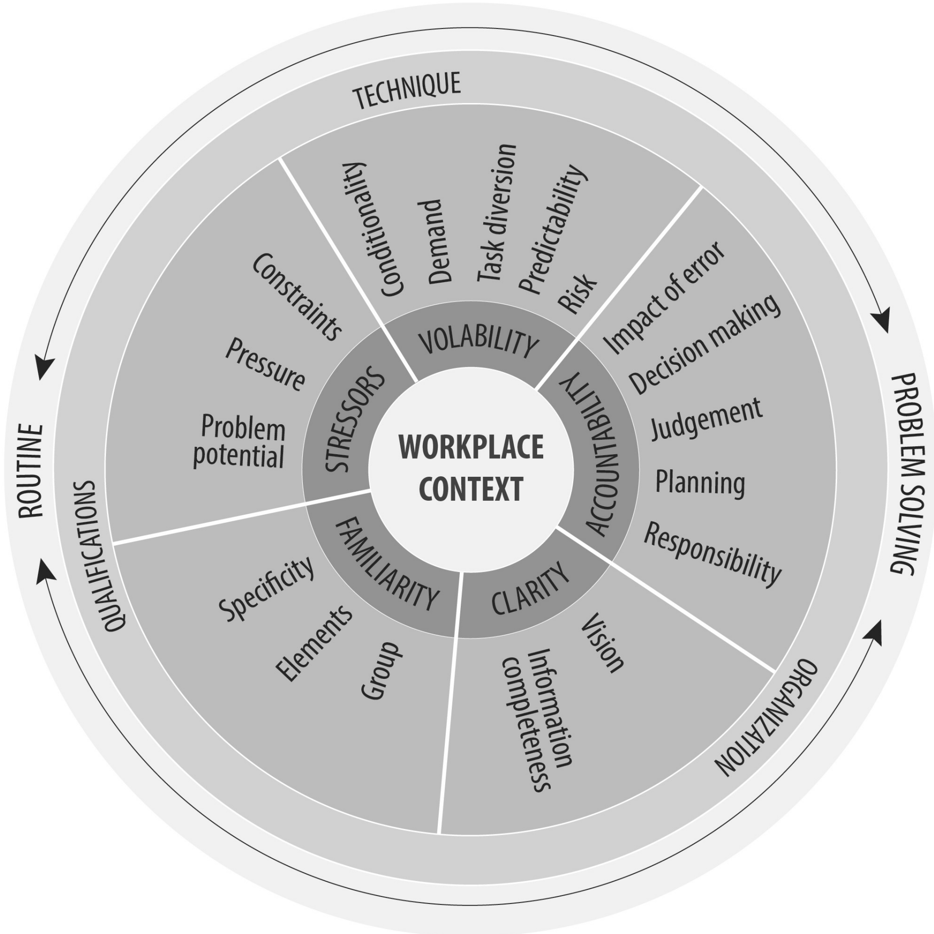


Figure 2. Elements that influence the context from Keogh et al. (2014) and Wedege (2002) combined.

If all the elements presented by Wedege and by Keogh et al. are taken into account, one realizes that the success of a task, whether it is a problem-solving situation or not, does not rely solely on the mathematics in use, but on the choices made throughout the process (Straesser, 2015, p. 673). These choices can be related, for instance, to the use of different procedures, techniques, timing, tools, the mood and skills of the person executing the task or solving the problem, etc. Ultimately, the choices can be related to the use of mathematical knowledge as well. In Figure 2, I combined Keogh et al.'s and Wedege's elements in a compendium of what can be offered in a workplace context. Each element plays a unique role and influences the development of the work itself (see Figure 2). None of them occur in isolation and their dependence is varied, determined by the context and the personal traits of the people involved. Recent studies have shown that both contextual and individual elements influence numeracy proficiency and that numeracy is used more at work than outside (see OECD, 2015; Duchhardt, Jordan & Ehmke, 2017; Jonas, 2018). The level of numeracy at work changes according to the individuals' expectations, resources and their personal disposition (see Jonas, 2018). The interrelation between the workplace, numeracy and the everyday use of mathematics is directly connected to the level of engagement of the individual; but at the same time, many adults' mathematical skills are deficient (Duchhardt et al., 2017), and often less complex mathematical knowledge is used compared to the use of advanced knowledge. The use of mathematics lies between the numeracy of the individual and the mathematical requirements of the work. Research indicates that more investigation directed towards assessing the mathematical requirements of different jobs is needed (Duchhardt et al., 2017).

Figure 3 is my own overview of the everyday use of mathematics, comprising its use at work and outside work. One of the first critiques of Figure 3 would be to question where the formal context of mathematics would be located (i.e., school mathematics). I intentionally avoided placing it because, given that the use of mathematics is a vast field, this thesis is specifically directed towards work. Therefore, the first division of everyday use of mathematics presents only two reciprocally exclusive options: at work and outside work (see Figure 3).

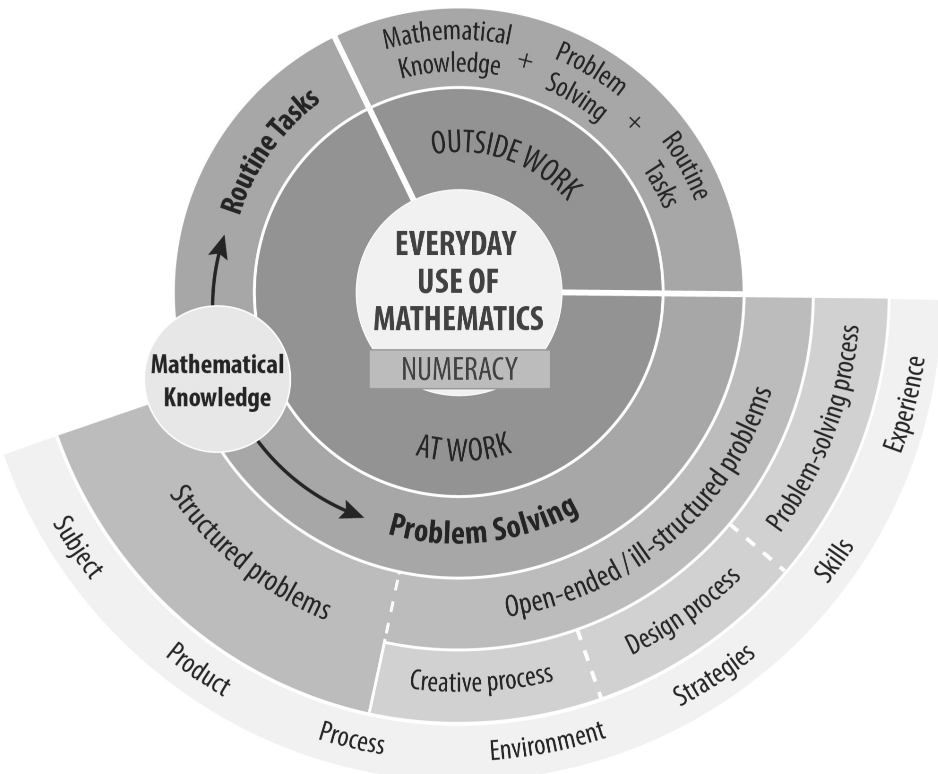


Figure 3. Everyday use of mathematics framed in a dichotomy at work and outside work.

Both fields are held by the ring of numeracy and both abide by a combination of mathematical knowledge, problem solving and routine tasks. These three components entangled constitute the everyday use of mathematics. Mathematical knowledge plays a central role as it may be considered a tool for executing both tasks and problem-solving situations. Routine tasks require repetition and experience. The problem solving component introduces a more complex part. In many workplaces, problem solving is an expected situation, and becomes part of the routine to some extent (Fitzsimons & Boistrup, 2017). In other words, facing a problem is part of the daily activities: e.g., when a client places an order, especially if it is a tailor-made product, task or job. However, in Figure 3, I use the dualistic understanding that Jonassen (2000) offers of ill-structured against structured problems (see Figure 4 p. 29 for further development of the concept of problem solving). The first category, also called open-ended problems, are where different solutions are valid and divergent thinking is fostered (Guilford, 1950; Becker & Shimada, 1997; Acar & Runco, 2012; Basadur, Gelade & Basadur, 2014). For structured problems, the combination of knowledge, strategies, skills, and experience will lead to finding a suitable solution. With open-ended problems, the solution may or may not require innovation. If it requires innovation, it becomes a creative process, and if the solution entails attribution of new features, the process becomes a design process. The concept of problem solving is reviewed in more detail in the following section.

2.4 Problem Solving

The conceptualization of the term problem solving changes drastically when considered within a formal education setting or elsewhere (e.g., at work, at home). In an educational setting, problem solving could be considered as a provoked and structured situation meant for a person (i.e., the learner) to develop a concrete ability or abilities where, most of the time, the use of algorithms is sufficient (Tall, 2013). On the other hand, outside an educational setting, problem-solving situations are neither designed nor structured. Problem solving is a spontaneous situation in which the subject does not know how to proceed, resulting in a gap between the current situation and the desired situation (Schoenfeld, 1983; Bodner, 1987; Tall, 2013). Subjects bestow a problem-solving situation with different degrees of difficulty according to their experience, practice and knowledge, since finding a solution that is obvious to one person might be complicated for others and demand more effort from them. Factors such as experience, familiarity and the conditions of the context in which the problem solving is sited influence the process of finding a solution or solutions.

Jonassen claims (2000) that problem solving contains two traits that are essential. The first is the unknown component, and the second one is that the action of "solving" must have some value attached to it; in other words, there must be some reason to find a solution. Yet, to conceptualize and define problem solving is far from straightforward (Jonassen, 2000, p. 65), since problem solving is not homogenous in any way (i.e., form, content or process). There have been numerous studies in the field of problem solving (see Törner, Schoenfeld, & Reiss, 2007; Toy, 2007; Schoenfeld, 2013; Viitala, 2018; Mamona-Downs & Downs, 2005), many of them focused on finding new strategies and heuristics (e.g., Polya, 1945; Mason et al., 1982/2010; Basadur et al., 2014) and others focused on problems set in realistic contexts and ill-structured problems (e.g., Pozzi, Noss & Hoyles, 1998; Jonassen, 1997; 2000).

In my attempt to describe my understanding of the process, I have adopted Jonassen's (1997) approach, where problem solving is described as either well-structured or ill-structured. In Figure 4, I display the elements I consider essential to problem solving. Ultimately, figure 4 could be viewed as an extension of the element problem solving in Figure 3. However, I considered that due to the significance of the concept problem solving for this thesis, it deserved a separate figure.

Well-structured problems are those with a clearly defined initial situation, involving concepts from the same knowledge domain and with precise and exact answers and solutions. Often, well-structured problems require specific processes to be solved, and these problems are artificially situated, like those at school (see Jonassen, 1997; Toy, 2007). On the opposite end, ill-structured problems emerge from a specific context and are context-dependent (Llorente, 1996). According to Toy (2007), most everyday life problems tend to be ill-structured and open-ended, and therefore may yield to various approaches, with no concrete algorithm and with more than one suitable solution. Solving ill-structured problems requires a different set of skills and strategies (Belland, 2013; Lin & Lien, 2013).

In Figure 4 the side of ill-structured problems is expanded as my work is oriented towards those situations emerging from the daily work. Ill-structured problems leave room for divergent thinking. Guilford (1950), while studying the intellect, pointed out the divergent unconventional production of ideas. Further studies on divergent thinking indicate that expertise is germane to divergent thinking, sometimes with positive influence and in some cases as a drawback (Acar & Runco, 2012). The results of the studies of Acar and Runco also indicate that divergent thinking is strongly connected to the generation of ideas, but not so much to the implementation of ideas (Acar & Runco, 2012). I consider the implementation of ideas to be a crucial part of problem solving (this idea will be developed later). As Figure 4 shows, divergent thinking and open-endedness are the crucial elements for ill-structured problem solving.

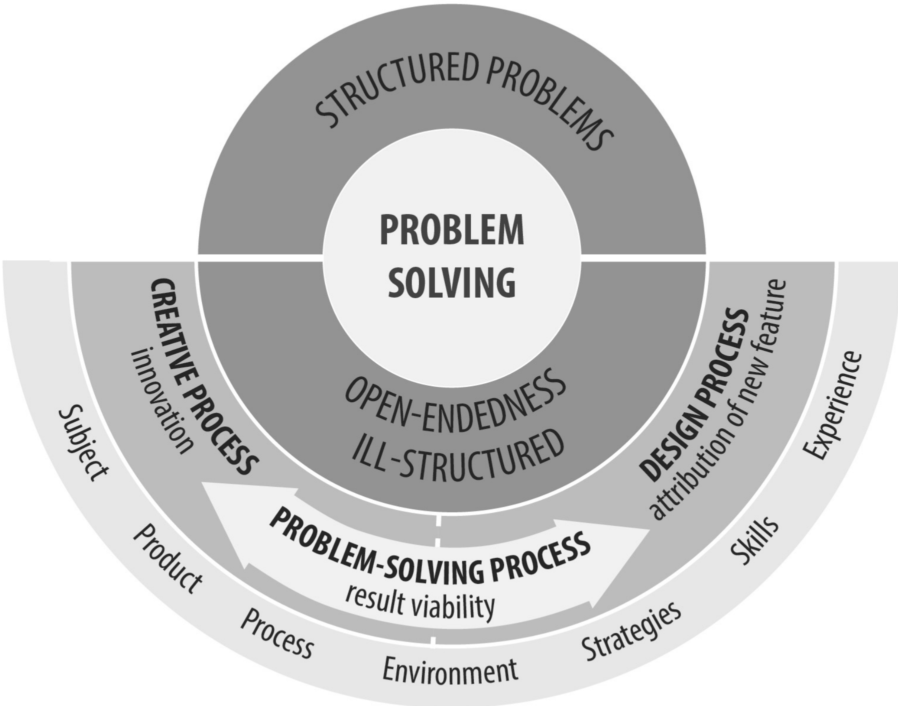


Figure 4. Problem solving

However, depending on the input of these two elements, the outcome could be placed under three different categories. One category would be a creative process, i.e., when the innovative traits of the product or the process are significant. Another category would be a design process, where the outcome gains new features or traits, but not necessarily an innovative identity. The third possible category would be a problem-solving process where the creativity and design of the product or process are trivial and irrelevant to the purpose. Most ill-structured problem-solving situations at workplaces are not expected to have innovative solutions with a creative component, but rather practical and viable solutions. The three outcomes are not exclusive, and they can be interrelated (this will be discussed later, as the relation between them was a part of the studies for this thesis). In Figure 4, the outermost ring contains vital components of the ill-structured problem-solving situation, regardless of their type. The subject, the intended product, the environment, the process of solving itself, the strategies and skills and the experience shape the totality of the ill-structured problem.

Whether well-structured or ill-structured, Jonassen also considers problems to be defined by their domain (i.e., concepts, rules, principles), by type (e.g., ill-structured problems can be design problems, dilemmas, troubleshooting, etc.) or by process, which at the same time depends on the solver’s knowledge, skills and understanding of the situation (1997). The exact process of solving a problem varies depending on the type of problem that is being faced; however, there are some steps that are generally taken when solving any problem (Jonassen 2000; Belland, 2013). The first step is to represent the problem by defining what is happening, creating a detailed mental representation of the current situation and possibly the desired one. The next step is to determine what knowledge is needed to solve the problem and to find such information. Since ill-structured problems are open, this step relies fully on argumentation, since there are no guidelines on what is needed in terms of knowledge. Argumentation implies forming ideas and reasoning to justify and reach conclusions to support a claim (Jonassen, 2000). The third step of the process of problem solving is to develop a solution and test it. Argumentation is also essential in this step as there might be different solutions and different points of view dependent on the solvers' epistemic beliefs (Jonassen, 2000; Belland, 2013).

2.5 Creative Process

The previous section on problem solving unavoidably leads me to examine creativity and the creative process. When a solution to a problem is found effortlessly, it cannot be deemed a problem, but rather a routine task (see figure 2, p. 25). Therefore, problem solving requires a bit of creative input, something more than experience and practice can solve (Liljedahl et al., 2016, p. 6.; Vidal, 2009, p. 417). However, I argue that this insight cannot convert the entire process of problem solving into a creative process. During the research process for this thesis, I encountered several different papers, articles, studies and books that offered, maybe unintentionally, the concept of problem solving as a synonym for invention, creation or the attempt to discover something (e.g., Amabile, 1983; Lubart, 2001; Leikin & Pitta-Pantazi, 2013). Lubart justifies this through observing that, in many studies, the term problem solving is used to refer to a task that a person tries to realize (2001, p. 297). However, this definition of the term does not correspond to the understanding of a situation in which one does not know how to proceed, but simply to an activity or task. The polemic is served. This is, for example, the case with Hadamard (1945). The necessary reading of his book *The Psychology of Invention in the mathematical field* clarified certain aspects of the process of invention but at the same time generated reservations about the equivalence that his work seems to expose, when it uses the term problem solving as a synonym for invention:

> Before trying to discover anything or to solve a determinate problem, there arises the question: what shall we try to discover? What problem shall we try to solve? (Hadamard, p. 124)

Hadamard continues describing two different types of inventions. The first type is when the procedure is unknown (where the unknown is how to reach a concrete goal), and the second type is when, after having created something, one should figure out a use for it. Hadamard remarks that mathematicians deal mostly with the second type. Within my work, the first type seemed to fit the case of problem solving. His stages of the invention process led my readings and theoretical framework to continue towards the creative process. The controversy persisted throughout my research.

Creativity is a complex concept to describe, and it has been studied from different perspectives and disciplines (Guilford, 1950; Amabile, 1983, 1988; Lubart, 2001; Vidal, 2009; Herman & Reitel-Palmon, 2011; Isaksen, 1995; Mumford, Medeiros & Partlow, 2012). There are several parameters to study within the concept of creativity, including (as Figure 4 displays) the subject, the product and the process (Amabile, 1983). Regarding the subject, Vidal (2009) differentiates between three creative subjects. One of these is the problem solver, who tries to solve a problem in a creative manner. The second type is the artistic individual who creates new pieces of art. The third creative subject embraces creativity as a lifestyle everywhere. Considering the product, Vidal (2009, p. 415) also claims that innovation and creativity within a product can be called design (see section 2.6. Design Process). Amabile conceptualizes creativity and claims that:

> A product or response will be judged as creative to the extent, that a) it is both a novel and appropriate, useful, correct or valuable response to a task at hand and b) the task is heuristic rather than algorithmic. (Amabile, 1983, p. 360)

Interestingly, this definition does not include the term problem solving and the traits emphasized are novelty and appropriateness. Amabile defines two types of tasks: the algorithmic ones, with a clear purpose, and the heuristic ones, some with a clear purpose and others without needing the subject to define the situation. With the latter, problem discovery becomes a relevant part of the creative activity (e.g., as cited in Amabile, 1983, p. 360: Campbell, 1960; Getzels & Csikszentmihalyi, 1976; Souriau, 1881). A task may be algorithmic or heuristic depending on the subject's knowledge and experience and the context. On the other hand, if a task is resolved using an algorithm, it means that the algorithm had already existed and therefore the task would be a well-structured problem (see Amabile 1983; cf. Jonassen, 1997).

Amabile also presents a model for the creative process for individuals and small groups, which is very similar to the model proposed by Wallas (1926). Wallas' model, later developed by Hadamard (1945), consists of four stages: preparation, incubation, illumination and verification. Amabile's model contains the equivalent: task/problem presentation, preparation, response generation and response validation and outcome (Amabile, 1983, p. 367; see Appendix B), yet all dependent on three external components: task motivation, domain-relevant skills and creativity-relevant skills. I believe that Amabile's model does not address a crucial element: the context, which conditions and influences the key components. The stage model has been criticized and modifications and improvements presented. For instance, some research claims the existence of a phase prior to the preparatory stage, when the problem or the situation is acknowledged and gaps are found (as cited in Lubart, 2001: Amabile, 1996; Getzels & Csikszentmihalyi, 1976; Osborn, 1953). Also, Sapp (1992) introduces the frustration phase, which might appear between the incubation and illumination stages.

Vidal states that creativity is connected to divergent thinking as it is the ability to see a situation in various ways (i.e., divergent thinking), trying to foresee a new perception or a different point of view (2006; 2009). According to Vidal, creativity employs different tools in its process, such as fluency, flexibility, originality and elaboration. He presents fluency as a means to deliver quickly in a knowledgeable way. One tool to develop and expand fluency is brainstorming, which is a way to engender ideas (Osborn, 1953; cited in Mongeau & Morr, 1999). Vidal explains that Osborn's intention was to separate idea evaluation from idea generation, since brainstorming seems not to be beneficial for complex situations, as some of the ideas generated might not be good enough (2009). However, I find this claim to be frail since, among all the generated ideas, even if some are weak and others incorrect or not suitable, some may provoke a chain reaction to associate to further ideas closer to the one that eventually becomes a solution. Vidal recognizes flexibility as the ability to manage ideas and proceed in different ways, i.e., adapting. For instance, within the creative process, flexibility as a tool allows, in the case of frustration, a change of route and the opportunity to integrate new ideas and alternatives. Vidal also mentions provocative questions aimed at broader thinking and directed to aspects that might not have been considered before. Some might be nonsense and others elicit thinking out of the box. He describes originality as the tool for leaving the obvious and common aside in order to pursue the unusual and unfamiliar.

2.6 Design Process

Design is closely related to creativity. Owen (2007) argues that creative individuals work either via discovery or via invention. He identifies the first group as *finders* and the second group as *makers*, also called *designers*. The goal of design is not creativity but the construction or production of a new distinctive artefact or product considering the needs, the purpose, the circumstances, and the outline (Rosenman & Gero, 1993). In other words, design requires that something be produced, not just abstracted. However, when it comes to the first production of a new artefact, knowledge and skill work hand in hand. Namely, the subject who is producing needs to have the ability to create and materialize the artefact by hand, having experience and knowing the materials (Risatti, 2007). This description distinguishes them as craftsmen (Risatti, 2007; Sennett, 2008) but at the same time as designers, since they have an acute sense of form and shape and they aim to produce several units.

Inventiveness in design and crafts appears to be shaped and limited by factors such as being client-oriented and context dependent. Roy (1993) claims that design often starts from the personal experience of the designer in an attempt to serve the needs of the user or client. I argue, based on the data of the study, that the creation of an artefact by the hands of a craftsman starts off the same way. Designers might use a mixture of 2D sketches and 3D models to adjust, improve and develop the product (Goel, 1995; Dorta, Pérez & Lesage, 2008; Pei, Campbell & Evans, 2010; Lahti, Kangas, Koponen & Seitamaa-Hakkarainen, 2016). The sketches and drawings help in the refinement of ideas, shapes, or the proportions of the product in the early stages of the process (see Goel, 1995; Lawson, 1997; Aspelund & Kontzias, 2006; Cross, 2011). The physical artefacts (e.g., prototypes or mock-ups) might be seen as the final product for a craftsman; however, for a designer they are the test model for what will become a mass-produced product (see Risatti, 2007; Temeltas, 2017). Lloyd and Oak (2018) state that the design process links the present with the future and becomes the arena in which different stories are connected. At times, those stories commence from the designer; at others, they tie to the product. The story that the subject explains may add structure and meaning to certain modifications or traits of the product, in addition to value (Lloyd & Oak, 2018). A designed product may be considered creative if it exhibits new features, traits or value. Rosenman and Gero claim that these aspects are fully subjective as well as related to time and place, given that the creativity of a product fades away with time as the product becomes ordinary and common (1993). Rosenman and Gero describe four different design procedures, i.e., mutation, combination, analogy and design by first principles (1993, p. 126). Before any of these procedures, the subject must establish if models or prototypes that possess the desired requirements and features already exist. Through mutation, some of the elements of an existing product are modified to meet the new requirements. Combination blends elements, models or the configuration from different existing products. Design by analogy associates traits, ideas or elements from different existing products and reconnects them in a new shape. Design by first principles starts from the identification of the requirements and details desired and searches for a suitable arrangement and configuration (see also Cross, 1997). However, these procedures do not need to appear individually and may be blended and combined during the design process. According to Brown (2008, p. 4), the design process undergoes intangible spaces of inspiration, imagination and implementation along with different modes in which the designer is immersed. The modes are approaches to dealing with the task of designing: empathizing; defining; ideating; prototyping and testing (see Stanford d.school, 2013). The modes are not lineal and may appear in altered order. The designer *empathizes* by observing and becoming engaged with the addressee of the product. Designs are not meant for general public but for a specific audience and thus require specific values and traits to be conferred on the product. By analyzing the findings of the empathizing mode, the designer *defines* his own principles. The define mode is close to the *preparation* phase in the creative process, since it guides and prepares the efforts of the designer (see Amabile, 1996). In the ideate mode, where flexibility and fluency are essential, the approach is detailed, ideas generated and alternatives to the design suggested. The next mode is the prototype mode, when the ideas are materialized or embodied (Goldschmidt, 2017). A prototype is an experimental model of a product used to corroborate and prove the ideated product and its details, and, if needed, make improvements (Goel, 1995; Aspelund & Kontzias, 2006; Brown, 2008; Cross, 2011). Prototypes exemplify the paradigm of learning by doing. Prototypes are typically constructed quickly and cheaply, since their purpose is not perfection but a sample for evaluation and consideration if the product meets the goals. Throughout the process of design, the need to build a prototype is common, since the designer then gets to try, play and experiment with the product to assess the state of the product (Isa & Liem, 2014; Stanford d.school, 2013). The last mode is the test mode, when the prototype is placed in context. These last two modes seem equivalent to the *verification* phase in the creative processes.

3 RESEARCH TASK

The aim of this research was to examine problem solving and the mathematical knowledge used at work. Two different settings were explored (i.e., a farm and a woodwork workshop). The researchers looked for the participants' perspectives regarding the use of mathematics at work and the problem-solving strategies they used. The research was developed through three main objectives. The first was to investigate a natural setting where the use of mathematics was not obvious, emphasizing the processes that farmers were involved in by analyzing different situations. Then, in a setting where mathematics was more obvious, the workshop of a cabinetmaker, the second objective was to find out what kind of mathematics was used in their everyday work and how problem solving and finding solutions to emergent problems were intertwined. The third objective was to shed light on what role problem-solving processes played in creative and design woodworking processes.

The main research questions behind this dissertation were: what are the mathematics that farmers and cabinetmakers use at work? What are the problem-solving situations that farmers and cabinetmakers encounter in their daily work and how do problem-solving situations influence other ongoing processes? The research task was developed through three different studies (Study I, Study II and Study III) and, therefore, the questions were modified and adapted to the cases handled in each of the studies as follows:

I	RQ1	What mathematics do farmers use in their everyday work?
II	RQ2.1	What is the mathematics used by cabinetmakers at work?
	RQ2.2	What problem-solving situations do cabinetmakers face at work typically?
	RQ2.3	How does the problem-solving process proceed?
III	RQ3.1	What is the cabinetmaker's process of designing and creating a table?
	RQ3.2	How do the problem-solving situations influence the design and creation of a table and what is the role of the jigs within the design process?
	RQ3.3	How are the problem-solving, design and creative processes intertwined?

Table 1. List of research questions from each study.

4 OVERVIEW OF THE ORIGINAL STUDIES

This section provides an overview of the three studies, including their aims, methods and findings. The articles that follow at the end of this dissertation contain more detailed descriptions of the context of each study, as well as the specific methods used. In addition, in Chapter 5, I describe in greater detail my methodological positioning within the approach used. I uncover some variations in terminology, such as the descriptive approach used in Study I in contrast to the ethnographic approach used in Studies II and III. These variations in the studies are a sign of how the research process evolved.

4.1 Study I

Farmers do use mathematics

The goal of Study I was to identify the mathematical knowledge used in non-formal contexts, outside the scholarly environment, since mathematical knowledge is found in many places. Study I presents findings from the use of mathematics in the context of livestock farming, specifically that of rearing calves. In conjunction with mathematical knowledge, there is adult numeracy, which depends directly on the person's common sense and the context (Fitzsimons, 2008; 2013; 2013a). The theoretical framework of Study I departs from the connection between the concept of context and the everyday use of mathematics (Gainsburg, 2005). The context used in Study I is a calf-rearing farm and two farmers participated in the study, Jaume and Elena. The methodological approach was an ethnography and the main idea was to observe the participants within their own working context and to perform unstructured interviews to explore the mathematical content of their daily activities. The study focused on the primary obstacles detected in the everyday life of the farmers i.e., the problems encountered by the farmers before their daily activities had become routines. Jaume and Elena did not seek solutions via pencil and paper strategies, but their strategies were contextualized and limited by the farm settings.

Study I presents two problematic situations at the farm where the farmers' use of mathematics is apparent. Both are related to feeding the animals. In the first situation, Jaume had to distribute the space in a barn to organize the feeding of the youngest calves. The second situation was focused on the use of different objects as measurement tools. For the first situation, the study describes how Jaume had to optimize and redistribute space in a barn to feed 50 calves. The data indicated that Jaume had unlimited solutions available, but he wanted to keep it simple. He created different models of fences before finding an adequate one. He used trial and error by positioning the fences in different ways to find the most suitable and convenient division. He also divided the problem into smaller parts and had different aims for each part. The first aim was to have a system that would allow differentiation and separation of the calves that had been fed from those that had not. The second aim was to have an area where the animals could eat in peace at the same time in an orderly manner. The third and final aim was to obtain the maximum amount of space for the animals to roam about while they were not being fed. Jaume's solution was to build fences to separate the fed from the non-fed animals, creating a sub-space at the same time where the feeding would take place. Another solution was for Jaume to make the fences movable. Jaume designed and welded the fences himself with simple geometrical shapes, mostly rectangular, and placed them so that when the animals needed to be fed, there were three spaces available: one for those animals being fed, one for those already fed and one for those waiting to be fed. After the feeding Jaume would remove the fences and thus the animals would get the maximum space available to roam about. The solutions Jaume devised for the redistribution of space in the barn showed his capacity to reason mathematically. Jaume reformulated the problem to examine different possibilities and alternatives, rejected and verified options, described and gave arguments for his choices and finally found solutions. All Jaume's actions were aligned with Polya's problem resolution (1945).

The second situation presented in Study I appears in the collected data in a variety of forms, e.g., when Jaume talked about feeding the animals, he mentioned a "pot of milk" when addressing the amount of milk fed to a calf. He had standardized the measurement to a "pot of milk". Jaume and Elena did not use conventional instruments since these were not available at the farm. Consequently, the absence of measuring tools or utensils provoked the search for alternatives available in the given context, the farm. The farmers created, and used as resources, objects that were not designed for that use, and consequently often had to be modified and adapted for the new use (i.e., artefacts, see Verillon and Rabadel, 1995). In Study I, the interviews with the farmers indicated that their conception of the use of mathematics was related only to paperwork and animal identification numbers. The data collected suggested differently as the mathematics in use could be found in many other places. The findings in article I can be grouped into three mathematical domains: the measuring domain, the numerical and quantitative reasoning domain and the geometrical-spatial domain. The measuring domain included, e.g., calculations of milk and medication doses or volumes of pots and containers for executing their daily tasks. The numerical and quantitative domain covered counting animals when animals were being fed or vaccinated or the amount of food needed. The geometrical-spatial domain included distribution of the barn space and different arrangements of fences while feeding, vaccinating animals or cleaning the spaces. The data in Study I shows that both farmers think intuitively and use common sense (Coben, 2000). The context of the farm shapes their responses and solutions to the problems they face, as well as their daily activities. They make sense of their environment and use contextualized mathematical knowledge.

4.2 Study II

Cabinetmakers' Workplace Mathematics and Problem Solving

The main goal in Study II is to clarify what type of mathematics Finnish cabinetmakers use daily at work, as well as how problem solving and finding solutions become intertwined. Study II focuses on the mathematics in use and problem solving at work, since in order to solve problems at work, the knowledge must be contextualized and adapted to the circumstances of the context. Study II briefly describes the different ways of understanding problem solving according to Liljedahl and Allen (2013), as well as the creative process that Hadamard (1945) adapted from Wallas (1926) with the stages of initiation, incubation, illumination and verification. In terms of methodology, Study II focuses on four cabinetmakers at their workshops. The data was collected in three stages through an ethnographic approach and consisted of workshop observations, fieldnotes, interviews, videos and photographs. The data analysis revealed that the problem-solving situation emerged primarily when the cabinetmakers had to build a jig. Jigs are self-made tools created to support and guide other machines or to hold together different pieces during a task (Paavola & Ilonen, 1981).

The analysis of the data in Study II was inductive and qualitative. The interviews were transcribed, and different themes were identified. The data was classified under three main topics: mathematics, problem solving and jig creation. The findings in Study II are presented in two different parts. First, a description of the cabinetmakers individually. Jacob is a man who enjoys working with his hands, a perfectionist who used a lot of time on his projects, shared ideas and consulted others, preferably individually. Thomas is a traditional cabinetmaker when it comes to methods and declared that he loved mathematics but refused to use advanced mathematics (such as trigonometrical reasoning) and computers in his daily tasks. According to Thomas, basic mathematics, along with the trial and error method, could be used everywhere. He is a social person and appreciates a break to chat with his workmates. Anthony is patient, serene and wants to experiment with other materials such as different metals. He feels comfortable using computer programs to design items and make calculations. Frank is a calm man and likes to discuss his work and methods with others who have more experience. He feels uneasy using technology.

The second part of the findings in Study II is presented from the point of view of the themes that emerged during the analysis: mathematical knowledge in use, problem solving at work and problem solving as a creative process. Within the theme of mathematical knowledge in use, the cabinetmakers clearly identified the use of basic operations to e.g., measure, cut, construct joints, glue or make holes. The data of the study also shows estimation of time and quantities, percentages and proportions, along with 3D projections, outlines and sketches. Basic notions of geometry were used daily to calculate diameters, areas, perimeters, volumes and to make different transformations. Calculation of angles was done to facilitate adjustments in the machines to obtain perfect cuts. The mathematical knowledge of the cabinetmakers was put to the limit, but not all of them used trigonometry. Some considered it essential to their work, others used alternative methods such as trial and error to avoid it, due to convenience or ignorance. The data revealed that it was possible to manage at work with basic knowledge of mathematics, but that it was mostly more time-consuming. The mathematical skills of the cabinetmakers were a factor that could limit their work. However, the mathematics in use was also limited by the fact that the material in use, the timber or wood, is a living material and its properties do not respect the precision with which mathematics operates. Regarding problem solving at work, the data indicated that the most challenging part of their work was when a problem occurred, and the work or procedure were interrupted due to a lack of knowledge and the worker did not know how to proceed. Most of these situations arose when the cabinetmakers had to build and/or create a jig. Jigs cannot be manufactured industrially since the measurements are different for each task and so is the accuracy and precision needed in each case. Some jigs, with experience, can be built immediately but others require time and are delicate. Due to the measurements and accuracy required, jig construction and creation becomes mathematical. In Study II, an example of this accuracy is described. Jacob had to drill a hole underneath the top plank of a table to attach the legs. The hole was one of the parts of a dovetail joint and therefore it needed a precise width, length, depth and incline. The jig needed had to support the router in the correct position to obtain a hole with the given measurements. Both, the jig and the hole were conferred mathematical attributes and thus they were mathematical. The efficiency of a jig was not always determined by the mathematical skills of the cabinetmaker, but, as Anthony mentioned, advanced mathematical knowledge can save time and effort, since precise measurements can be obtained without the delay that the trial and error method often requires. The cabinetmakers identified different strategies and steps in their problem-solving situations. For example, they examined past personal experiences as well as the experience of their work colleagues or tried various approaches and modified them when needed.

The third theme was the similarities and differences between problem solving and creative processes. The strategies and steps revealed by the cabinetmakers seemed to be analogous to the creative process stages of initiation, incubation, illumination, and verification that Hadamard described in 1945. According to the data of the study, both processes are different, but they are intertwined. Both processes are aimed at a final product. For the creative process, the product must be innovative, but on the other hand, for the problem-solving process, the main goal is the viability of the product, as is in the case with the jigs. Therefore, the cabinetmakers do not consider the jigs to be creative processes. Innovation is not essential in jigs, but functionality is. However, the cabinetmakers have different tasks and jobs where they need to create or design new cabinets or furniture. The study data revealed that those creative processes were intertwined with numerous problem-solving processes and often the success of the creative process relied entirely on success in different problem-solving situations.

4.3 Study III

Workplace Problem Solving within the Design Process

In Study III, the process of design and construction of the first prototype of a design table is documented with the permission of Matti, the cabinetmaker. The initial idea was to explore how the problem-solving situations influence the process. The design of a product can be creative or non-creative, and, for this study, design is conceptualized as the process of attribution of new traits to an existing type of product, a table. In this case, creativity is essential because, without creativity, the design would not succeed in the market (Howard, Culley & Dekoninck, 2007; 2008; Roy, 1993; Wimmer, 2016). Wallas (1926) distinguished four stages within the creative process and named them initiation, incubation, illumination, and verification. Sapp considered frustration an additional stage of the creative process (1992). Later, Amabile (1983, 1996) offered a model with different names for the stages: identification, preparation, response generation and validation and communication. Article III studies a specific job at a cabinetmaker's workshop and how the problem-solving situations (i.e., jigs) alter it. The study posed the following questions: what is the cabinetmaker's process of designing and creating a table? How do the problem-solving situations influence this process and what is the role of the jigs within the design process? How are the problem-solving, design and creative processes intertwined?

The study used a narrative approach to collect the data through interviews, informal conversations, and shadowing. The main focus was on Matti, the cabinetmaker who reported his experience and described in detail the process of creation and design of a table named Pekki, and the construction of its first prototype. Through shadowing, the study obtained a deeper understanding of how, when and why Matti's actions and doings took place and, specifically, recorded data regarding his motivation, mood, body language and work pace. There were recorded sessions, hundreds of photographs and fieldnotes. For the analysis of the data, the researcher and Matti made a list of all the steps documented in the process. To answer the first research question, the researchers made a general outline of the entire process of the building of the first prototype of the Pekki table, based on all the actions and documented tasks. All the steps were listed in order of occurrence. The photographs were organized by themes and placed according to the time they were taken within the process. The precise moments when problem-solving situations emerged were located. After this, and following the chronological events, the narrative of the creation of the first prototype of the Pekki table was initiated (Creswell, 2012).

Pekki's story is described in four phases. The first phase details how the idea of designing a table was conceived. For Amambile (1983), this is the identification stage and corresponds to Wallas' (1926) preparation stage. During the second phase, Matti envisioned the table's appearance. Matti intended to create something special using the timber available when a tree had distinct value for a customer. After months of thinking and trying to decide on the design came a period of frustration (Sapp, 1992) and the sudden moment the vision became clear after a trip to Japan. Amabile (1983) would describe the phase as response generation and Wallas (1926) would regard it as the illumination stage. Next Matti proceeded to build the first prototype of the table, at a 1:1 scale. He started by acquiring the timber and making the first cuts for the tabletop planks. Even during these first tasks, small problem-solving situations appeared when Matti had to build jigs. A total of eight jigs of varying complexity were needed in the process. In any case, all the jigs were adjusted to the measurements needed for Pekki. The final model of Pekki and the first 1:1 scale prototype differed a bit from one another. In Study III jigs were not considered creative processes. The need to create a jig generally appeared when Matti encountered a problem-solving situation (Saló i Nevado & Pehkonen, 2018); i.e., the need to build a jig emerged from the workplace tasks (Llorente, 1996). The data from the study indicated that jigs influenced the construction of the first prototype of the Pekki table, altering the time factor. The more difficult the jig construction was, the more time Matti seemed to invest. Some jigs were rather quick to construct due to Matti's experience. Others were related to trigonometry, but Matti decided to use the trial and error approach and thus spent more time on them. Another factor influenced by the jigs was progress. Jigs became the key to go from one step to another, and once a jig was built successfully, it could be re-used in other similar situations and therefore became a tool and part of the construction process. Jigs also influenced the precision in some of the steps of the process. Mostly, precision is mathematical and thus converts problem-solving situations into diagnosis-solution problems (Jonassen, 2000, p. 75). At this point, Study III diverges from Wimmer's (2016) idea that successful problem-solving situations are creative processes. The analysis of the data in Study III shows that some of the jigs did not influence the process at all and belong to what Howard et al. (2008) consider routine design. However, some jigs influenced both the creative and the design processes. According to the data, problem-solving situations are mediating processes between the design and creative processes. In the case of the Pekki table, the creative and design processes are parallel to each other and the level of creativity displayed in the problem-solving situations (i.e., jigs) was not relevant. Instead, the success of the problem-solving situations was significant, since the creative traits of the Pekki table depend on them and therefore they also have a direct impact on the design process. Jigs can be considered open-ended or ill-structured problems (Jonassen, 2000; Becker & Shimada, 1997) and they become mediating processes between the creative and design processes.

5 METHODOLOGICAL POSITIONING WITHIN AN ETHNOGRAPHIC APPROACH

In this section I introduce the settings and participants of this research. I discuss the ethnographic approach, the data-gathering methods used and the different types of data obtained. In addition, I address questions concerning my positioning in this research, the ethics of this research and the process of analysis.

5.1 Settings, Participants and Access

I decided to study the everyday use of mathematics, and thus, the closest *everyday* had to be something familiar and near. Firstly, I selected a farm and farmers from Lleida since it was a setting I was acquainted with (I lived for 20 years near that area during my childhood), yet not too familiar. Secondly, since by the time of the research I had moved on a permanent basis to Finland, I chose wood craftsmen and their workshops. Forest and wood are Finland's most valuable resources. Thus, cabinetmakers and carpenters are an important part of the wood industry in Finland (Finnish Forest Industries, 2019). Modern wood factories employ different professionals in the different manufacturing phases. However, the base of furniture production lies in the hands of craftsmen and their skills (Joyce, 1980).

The research took place mostly in the metropolitan area of Helsinki, but the data was gathered in the two previously mentioned settings: the first was the small farm from Lleida, Catalunya, Spain and the second data gathering setting was split between four cabinetmakers working in three workshops within the metropolitan area of Helsinki, Finland.

I was aware of the fact that gaining access to a workplace setting to conduct ethnography is complex since there might be constraints such as time playing against the researcher, as the working time of the participants is limited and their availability to answer the researcher's inquiries might be reduced (Smith, 2001). For the data collection at the first setting (i.e., the farm, Study I), I chose to study during my own holiday time, making sure beforehand that the daily practices on the farm did not differentiate a holiday from a regular working day since animals need to be fed on a daily basis, without exception. The fieldwork lasted a total of 20 days over intermittent periods during 2004. The time availability was a factor in my advantage as I was able to arrange the visits and fieldwork almost without restrictions. In the second workplace setting with the cabinetmakers, the interviews and visits to the workshops were agreed upon in conjunction with the cabinetmakers (Study II). The data collection period took place during 2010 and 2011, as well as during 2013. For Study III, I used shadowing as the main method to collect the data, along with video recordings, photographs and fieldnotes. I negotiated with the participant, Matti, to have access every time he had to work with the project. The data collection lasted 17 months during 2013 and 2014. This arrangement was a privilege due to the extreme flexibility needed from both sides: the participant (who committed to share his work and time on every occasion he initiated some sort of action with the product I wanted to document) and the researcher (as I needed to be available and within reach in a short period of time). To my benefit, in this case, Matti decided to work on his project during the evenings and afternoons, outside the usual working hours. This arrangement had its downsides as well, as the fieldwork recordings and data collection often ended late in the evenings.

Throughout the research I worked with different participants (see Table 2). For the first study at the farm, the participants were two farmers, named Jaume and Elena for research purposes, to protect their identities. When I contacted them to ask for their cooperation in this study, I provided them with a short but transparent explanation of the research and why I thought it was important. The first contact was uncomplicated and straightforward as I already knew them as family friends. I addressed them in Catalan, their and my mother tongue, as the use of that language helped me to gain access to information and to increase rapport. I specifically used the same dialect of the language (i.e., Català Occidental) and, accordingly as a native speaker, I was able to handle the nuances of the conversation and understand humor and irony when used (Kawulich, 2005). The access was granted without hesitation, and I had their understanding without having to negotiate much. In terms of the physical location, the farm was also a known place as I had visited it with my family during my childhood. However, I did not know it in detail in terms of a workplace and I showed my curiosity in my questions. Both Jaume and Elena were very eager to explain and share their knowledge with me. At all times, I felt welcomed and appreciated, and that they wanted to genuinely share their time, space, and knowledge. They seemed to forget right away that I was studying them. I chose the role of an ethnographer to relate to Jaume and Elena and to detach myself from overly familiar links, and it was easier for them to consider that I was doing work, wanting to know their ways and their work-related affairs. My approach was to appear naïve but at the same time knowledgeable, interested in detail and willing to ask questions and even more willing to listen to what they had to say and show (O'Reilly, 2009). But at the same time, as I was familiar with certain aspects or things, they did not need to start from scratch with all their explanations. I knew how to move around the farm, and I was trusted to do so freely without any time constraint. I was able to show up unannounced at the farm, participate in ongoing tasks or observe what was happening. However, as a courtesy and a sign of respect towards them, I agreed on some concrete meeting times and informed them beforehand of my visits. Sometimes we agreed on a specific timeframe when the workload would allow more discussions. This trust was an advantage in performing participant observations since they felt comfortable to give orders and to ask me to do things (about participant observation see O'Reilly, 2009, pp. 150-157). In addition, I occasionally stayed over late in the evenings, after the work with the animals was done. We ate and spent time together. This was a sign that I was welcomed and strengthened the trust between participants and researcher.

For the second part of the study, i.e., the cabinetmakers' work, I also used my personal contacts to ease entry. In this case, I knew one of the cabinetmakers, who became my gatekeeper to set up contact with the others, mapping out the social connections and easing the path to the other cabinetmakers (Kawulich, 2005). The contact was established gradually. Again, and for research purposes to protect their identity, the cabinetmakers were named Jacob, Thomas, Anthony and Frank (see Table 2). Jacob's interviews took place in his home and his workshop. Thomas, Anthony and Frank were interviewed at their workshops. They were not complete strangers to me, as I had seen them previously at some events related to my own social circles (Warren, 2001). This made gaining access easier and more comfortable for both sides. However, one of the cabinetmakers agreed to share the process of constructing a first prototype of a table, and through his cooperation and assistance in the process, he became an author in Study III (see Saló i Nevado, Pehkonen & Salminen, 2020). In addition to providing materials and data, checking my interpretations and suggesting changes, he gave full consent to use his real name, Matti Salminen, and to use all material collected during the documentation process, including disclosing his identity. Having considered the further implications and consequences that this validation could generate, I decided (along with the other co-author of Study III) to continue with the procedure as it established credibility, resolved and validated my understandings and analysis of the data (Bryman, 2004; Heyl, 2001).

	Participants	**Sites**
Study I	Jaume Elena	Farm in Lleida, Catalunya, Spain
Study II	Jacob Anthony Thomas Frank	3 workshops in the Metropolitan area of Helsinki, Finland
Study III	Matti	1 workshop in the Metropolitan area of Helsinki, Finland

Table 2. Research participants and sites.

The workshops, as the research setting scenes in Study II and Study III, were three workshops within the metropolitan area of Helsinki, Finland. One of them was located in a vocational school and used both for teaching and working purposes. It was spacious, well illuminated and equipped with all kinds of modern machinery. A second workshop was located in an old farriers' workshop with a combination of old and new machines and tools. It was used by several craftsmen for their own projects. Because it was not too spacious, the cabinetmakers took turns to work there. This eased the task of collecting the data as only our participant was present at the time of data collection. The third workshop was a rented space in a warehouse. It was well-equipped and spacious. Other cabinetmakers and small companies had their workshops in the same warehouse. In terms of contact with other professionals from the same field, this location and setting facilitated communication and networking with my participant.

5.2 An Ethnographic Approach Embracing Shadowing at Work, Participant Observation and Interviews

The focus of this research was the everyday work of the participants. I was not searching for isolated and surprising phenomena but everyday activities and details of their work. Thus, an ethnographic approach embraced the sensibility needed to reach my purposes, allowing access, among other things, to different procedures, routines, skills, rules, complexities, practices and strategies at the workplaces (Smith, 2001; O'Reilly, 2009; c.f. Beach, 2017). On the other hand, mathematical knowledge can be considered the result of human interactions in a concrete context as, for example, a workplace (Zevenbergen, 2000, p. 210) and again, an ethnographic approach permitted an interpretation of the actions and practices of the participants along with the mathematics in use and the problem-solving situations. This type of approach allows the retrieval of experiences and details that would be unattainable to the researcher using other research approaches. However, ethnographic studies tend to focus on a specific place in a static way (see Trouille & Tavory, 2019) and problem solving is not a static part of mathematical knowledge. The open-endedness and the required flexibility in problem solving are completely subject-dependent and thus, in continuous motion, non-static and therefore more difficult to capture and study. Among the ethnographic methods I used, shadowing provided access to another level, generally emotionally delicate and complex to gain access to (Trouille & Tavory, 2019). Shadowing contrasts with ethnographic observation as it is done across situations and settings, having a greater ability to connect with the meaning, the settings and the subject. It is mostly used in qualitative studies of individuals in their work context, as in my case (McDonald, 2005; Czarniawska, 2014; 2014a). Shadowing allows the researcher to keep pace with the events or happenings as they occur in real time and over different spaces, making mobility a key element (Czarniawska, 2007; Vásquez, Brummans & Groleau, 2012; Gill, Barbour & Dean, 2014). At the same time, shadowing goes beyond an ethnographic observation by enhancing the data with details about the body language, the pace or the participant's disposition. The challenging part of the method is that it requires a great deal of trust as the researcher follows the participant over long periods of time while asking questions and seeking clarification from them. Therefore, communication and cooperation are prerequisites for success (Gill et al. 2014). Shadowing places the researcher in a position of constant adjustment to the ongoing situation and demands flexibility from both the researcher and the participant. The researcher is supposed to encourage participants to reflect at the same time as the task is being performed (McDonald, 2005; Gill et al., 2014). I used shadowing to collect data in all the studies, but particularly for Studies I and III. In Study II, I shadowed Anthony and Frank on two occasions, while they were using some of the machines, mostly to understand how they were operated. In Study I, the shadowing periods were few. I shadowed different tasks that the farmers had to do around the farm, such as feeding and cleaning the barns. The shadowing sessions were combined with apprenticeship and participant observation sessions when I was asked to perform certain tasks to learn the procedures. Retrospectively, considering the methods used in this research, I see the shadowing at the farm for Study I as a preparation session for the shadowing done in the workshop for Study III. Shadowing in Study II was minimal.

In Study III, I followed Matti the cabinetmaker over the periods of time he was working on the prototype of a table. Shadowing was the main data collection method, and it was done over non-consecutive days. While shadowing, I asked questions to clarify various situations or the tasks that Matti was executing. The main purpose of those questions was to open up the meanings and nuances of those moments. During the shadowing, I wrote some short fieldnotes about what was being observed to capture the essence of those moments (Quinlan, 2008). Then, when each of the sessions was over, I recapitulated and organized the happenings of the evening within the whole process. Some sessions were more significant than others in terms of speed and advances in the process being observed, while others were slow in pace but with a lot of time used by Matti for thinking and ruminating in the search for the direction to follow. I used the shadowing technique to focus on two different elements: Matti and the prototype process. In most cases, shadowing is used to investigate a subject's role and ways of working (McDonald, 2005; Czarniawska, 2014a); but my objective during the ongoing process was to focus on the process at the same time as well, with a detailed log of the steps. For this purpose, I decided to video-record all the shadowing sessions with Matti. During the shadowing for Study I, I took fieldnotes, but did not record any video footage. In Study II, I mostly took notes and made a few short videos while shadowing during a couple of situations when the cabinetmakers were working with some machines at the workshop. In Study III, the video-recordings allowed me to relive and visualize what had happened and to notice once more the body language, time lapses and movements that I had observed in situ. The video-recordings were meant to support the other data collected through fieldnotes, observations, photographs, interviews and discussions.

In Study III, the shadowing was carried out in a workplace where there were only two people: Matti (the shadowed individual) and me (the researcher). This fact greatly decreased the disturbances and influence of other individuals that might be present in different workplaces (e.g., other co-workers, supervisors, clients, patients, etc.). In the case of shadowing on the farm, the situation was similar, as I mostly shadowed one of the farmers at a time (i.e., Jaume or Elena) and we were alone on the farm premises. Thanks to the fact that I was very familiar with all participants (i.e., Jaume, Elena and Matti), the shadowing sessions did not become awkward or odd. In the case of Study II, shadowing and participant observations were combined within the visits to the workshops supporting the interviews. Another fact that made the shadowing sessions casual and comfortable for all the parties involved was the time of the day. At the farm, I spent almost full days between 8.00 and 22.00. Matti's shadowing sessions were mostly in the evenings between 17.00 and 23.00. At the workshops with the cabinetmakers, the time was agreed on beforehand and therefore suitable for them. In principle, shadowing produces an overwhelming amount of data in the shape of notes and observations (Quinlan, 2008). For this reason, the video-recordings allowed me to focus the writing of my notes on those details not so easily captured by the camera and those sensations, impressions or thoughts that arose while shadowing.

Participant observation was the main ethnographic method that I used to gain access to the data at the farm (Study I). Participant observation enabled me to make a systematic description of the practices, artefacts and situations witnessed via observation and involvement in the daily activities of the farmers (Emerson, Fretz & Shaw, 2001; Delamont, 2004). My participation in some of their daily routines helped me to reach an understanding of the situations from the point of view of the farmers without disturbances. I participated in some activities, such as feeding the animals, cleaning the barns, morning and evening check-ups of the animals in different barns and feed preparation. Since I performed all these tasks with them, I had the opportunity to engage in interesting conversations related to the tasks and the work at the farm. I felt I was gaining their approval and at the same time increasing rapport. On the other hand, I also observed the farmers and kept some distance while they were performing other daily tasks to gain a detailed vision of what was happening (Emerson et al., 2001). The goal of my observation was to gain a complete understanding of the farmers' everyday tasks within the context of the farm without interfering (Emerson et al., 2001; Delamont, 2004; Kawulich, 2005; O'Reilly, 2009). In theory, both participation and observation could be understood as excluding each other, but for me, participation became a tool, an instrument to complement observation on the farm (O'Reilly, 2009). The conversations that happened spontaneously with the farmers during the participant observation procedures were very valuable for the study. However, it was imperative for me to listen, observe and maintain a receptive and open attitude, without prejudices (Emerson et al., 2001). The observation granted access to non-verbal expressions, feelings and emotions that appeared while collecting the data. In addition, I got the chance to verify some details in situ that had emerged during other ethnographic procedures or methods in use, such as interviews (Emerson et al., 2001). During the observations at the farm, I made fieldnotes and some audio recordings of the conversations that I had with the farmers. In addition, I collected visual data such as photographs of tools, spaces, and the animals to support the fieldnotes or the notes collected after the participant observations.

Interviewing was the main ethnographic method that I used to gain access to the data in Study II. However, it was also used in Study I and Study III to obtain complementary or background data (cf. Atkinson & Coffey, 2001). In Study II, asking questions and listening were essential elements of data gathering, as well as for developing a relationship of familiarity and trust with the participants (Heyl, 2001; O'Reilly, 2009; cf. Silvermann, 2017). Thus, at times, my interviews included a discussion over certain aspects, opportunistic questions or simply a conversation over a connected topic (O'Reilly, 2019, p. 18). The interviews, apart from being the main source of data, assisted me in establishing a stronger connection with the participants (i.e., farmers and cabinetmakers). The language used in each interview was the language the participants felt most comfortable speaking (i.e., Catalan or Finnish). English was used partly with the Finnish participants to clarify concepts as I am not a native Finnish speaker and I sometimes needed to make sure I understood what was being explained. I audio-recorded the interviews with a small personal recording device and I brought along paper and pencil as a backup (Warren, 2001).

For Study I, I interviewed two farmers, Jaume and Elena, regarding their everyday tasks, use of mathematics and problem solving on the farm. An initial interview to collect background data was semi-structured and the rest of the interviews were situated and acting out interviews. At the same time of the initial interview, I took some notes to back up and recall some parts of the interview. The situated interviews took place in different spaces of the farm, while walking around the different barns or while executing daily tasks. In addition, I used one of the barns as a stimulus for a stimulated recall interview, to engage Jaume in further discussions and details of how it had been distributed (Hodgson, 2008; Creswell, 2012). For the acting out interviews, the farmers showed their usual procedures and tasks. During the shadowing interviews, the farmers were followed while performing their work. When dealing with the apprenticeship interviews, the farmers were asked to teach me how to do different tasks. The interviews were transcribed into text files in Catalan, the language used.

The interviews for Study II were all semi-structured. Four cabinetmakers agreed to meet me and answer my questions regarding their everyday tasks at their respective workshops, the use of mathematics and the emerging problem-solving situations at work (see Appendix C). The interviews were arranged at a convenient time for each of the participants. The interviews were transcribed into text files in the language used (i.e., majority in Finnish with some parts in English). All transcriptions from both Studies I and II presented the dialogues and discourses of the participants (farmers and cabinetmakers), but no special attention was paid to pauses and other traces of spoken language. The pace of the conversations was slow and included laughs and jokes that I interpreted as the participants' signs of openness and inclusion that I needed as a researcher. For publication purposes, I translated selected excerpts from the interviews into standard English in all Studies I, II and III.

The first parts of the semi-structured interviews with all participants were meant to gain a sense of their background as well as the extent of their mathematical knowledge, the nature of their work and daily tasks and their personal perception of the mathematical knowledge in use. In those interviews, I listened carefully and respectfully to allow the participants to voice their views, opinions and points of departure by answering open-ended questions (Rapley, 2001; Roulston, 2008). My aim was to access their world on their own terms, avoiding leading questions that would condition their answers (Heyl, 2001; Frey 2004; Ayres, 2008).

5.3 Other Type of Data: Photographs and Video Recordings

Through the participant observation, the interviews and the shadowing, I obtained several types of data, such as fieldnotes, audio files, video footage, photographs, sketches, and drawings. Photographs were taken on the farm and in all the workshops as the visits were long and I wanted to capture details that I would otherwise not be able to retain. The photographs were mostly taken when the farmers or the cabinetmakers explained certain procedures or showed tools, materials or machinery related to the topics discussed. The photographs were meant to support the audio-recorded data and to recall instruments, tools in use, moments, situations, and spaces (see Appendix D). I used a digital camera or my phone. Both allowed me to take over 450 photographs at the farm, over 450 at the cabinetmaker's workshops and over 300 at Matti's workshop. Often, I took several shots of the same item or situation as I did not check the quality of the image while taking the photo and I wanted to make sure that the relevant details were captured. Some of the photos were blurry, unfocused, or moved. Most of the photographs were taken by me during the fieldwork periods except for a few photographs taken by Matti during those occasions when I was not able to visit the workshop. In order to protect the identity of the participants, the participants' faces were not photographed except for Matti, with whom I had a different agreement. At most, their hands appear in some of the photographs holding some tool or item.

However, photographs had another relevant role during the data collection for Study III. They were used for the stimulated recall interviews with Matti, the cabinetmaker. Matti was shown the photographs previously taken during shadowing sessions and he was asked to verbalize what had been transpiring in the photographs, which included the physical description, the thoughts and the feelings he recalled having at that moment (Lapenta, 2011; Bates, McCann, Kaye & Taylor, 2017). This was a way to gain an insight on his thoughts and thinking processes as he went about his work with the prototype. All the photographs used for the stimulated recall were taken by me during the fieldwork periods. The photographs were powerful since they had been taken while I was in the same place as the participant and being a witness of the same situation. The emotional charge of those photographs was intense, especially those showing moments when Matti was reflecting or ruminating over some detail of the process. Showing those pictures to Matti not only stimulated his memory in a way my questions could not, but also led to further discussions (Bates et al., 2017). He shared his own interpretation of the captured moments and the photographs gave him the opportunity to reflect on his own points of view. This was not the only time I used stimulated recall during the research. During Study I at the farm, I revisited some of the barns and used the locations as stimuli to recall different situations from the past, particularly related to problem solving. This proved to be a valuable approach to Jaume's narrative. However, this type of stimulated recall during an interview is not exempt from validity issues, as sometimes the participant might describe their present standpoint instead of recalling how it was in the past (Hodgson, 2008). Thus, the data gathered with the stimulated recall was contrasted with the participant observation data. The main difference between stimulated recall with Jaume and with Matti was the time factor. Jaume had to recall situations that had taken place a long time ago and when the researcher was not present (the stimulus was only the place). With Matti, the stimuli were the photographs, and the recall was done soon after the pictures had been taken and both of us (Matti and I) were present during the capturing process. This helped with the validity of the data.

Informed consent for taking the photographs was requested beforehand from all the participants regardless of whether the photographs included individuals or not. The case of Matti was different, since I agreed with him that all the materials produced while documenting the construction of the prototype would be available for him and he explicitly stated that I could take pictures of him during the process. Nevertheless, I made sure to inform how and where I was going to use the pictures. Specifically, I took pictures of him while working on the prototype, transporting and handling materials or reflecting over some detail. The stimulated recall interviews with Matti were researcher-driven as I provided the photographs for the interviews (Bates et al., 2017). Separately, I asked Matti's consent to use the photographs in the thesis summary. There were three occasions when I was not able to meet Matti at the workshop and he volunteered to document himself by taking pictures of what he had done. Before the next data collection session, we had a participant-driven interview where he was able to explain what had happened while I was not at the workshop, and he shared those pictures with me. The photographs used for the stimulated recall with Matti and their interpretation had a double objective. First, they were a realistic reconstruction of observations during the data collection. In other words, they were an exact two-dimensional record. Secondly, they evoked personal interpretations, responses, values and personal meanings from Matti (Lapenta, 2011).

In addition to the photographs during the data collection periods, I used video-recordings (i.e., audio-visual recordings). I made about 40 video recordings of different lengths. Some lasted a few minutes and others were up to 90 minutes long. For this purpose, I used my own digital camera recorder. I did not edit the material or manipulate them in any way, guided by scientific ethnographic principles (see Pink, 2007). I am aware that ethnographic knowledge does not automatically exist as observable facts (Pink, 2007) and that is why the primary role of my video-recordings was to support other data gathered through other ethnographic methods (e.g., interviews, fieldnotes, observation, etc.). The videos were meant to capture the real span of time of the tasks, and the conversations and interactions with the cabinetmakers. The visual recordings picked up the context and the settings to support my fieldnotes. Videos were used for data collection for Studies II and III. The videos made for Study II did not include the cabinetmakers in any way that they could be identified. These videos were short recordings, used as notetaking while shadowing in the field for procedures and tasks with machines or tools (Pink, 2007). Upon establishing contact with each of the cabinetmakers, I sought permission to make video-recordings (in addition to the audio-recordings) to get some footage about what was shown, and received their consent to use the material (Rapley, 2007). I also gave the participants concrete information about the purpose of the videos and how I was going to use them. In addition, I used video-recordings to collect data and document the construction of the prototype for Study III. The video-recording in Study III went further than mere video-recording of what Matti was doing to create visual data for analysis. Since I was documenting a process, I used the footage as a video diary of each of the steps. Along with it, I used a small notebook where I annotated hints, ideas, and thoughts that appeared while recording. The videos captured the complexity of the process, the physical behavior of Matti, his body language, his communication, personal comments and details of the workshop, machines and tools. Later, during the analysis of the data, the recordings allowed me to scrutinize the happenings as many times as needed (for example, while writing the descriptions for the narrative analysis). However, I am aware that some elements were left outside the lenses of the camera frame, because I had only one camera and it was carried by hand all the time from one side of the workshop to another. The workshop was composed of three rooms where different tasks took place. Some of the procedures consisted of meticulous use and movements of a tool against the timber and some others included repositioning big pieces of timber and adjusting large machines. I chose to position the camera mostly towards Matti's movements and actions, trying to focus on the activity that I was meant to document. Consequently, I combined the video-recording with other methods (i.e., photographs, fieldnotes, observation and interviews).

Video-recording raises the ethical issue of the identification of the participant, their privacy and the confidentiality of the data as exposed images might reveal (Gibson, 2008). However, this was not an issue with Matti as he gave his full consent to using the video-recordings and the photographs where he was shown for research and academic purposes. In addition, consent was revalidated at every step of the process (e.g., when writing articles, when preparing a research presentation for university or when using any of the data in the summary section of the thesis). In the case of the video-recordings for Study II, no images were captured in which the participants could be identified.

5.4 Ethical Considerations and Positioning Myself

According to the guidelines of the Finnish National Board of Research Integrity (TENK, 2019), the foundation of research with human participants is trust in the researcher. This trust can only be achieved by fully respecting the participant's dignity and rights. To accomplish this, I familiarized myself with the chosen participants, their community and their circumstances at work, as well as the moment they were living. In my study, all participants consented to participate voluntarily via verbal agreement (in some cases the consent was recorded in the form of audio files). Their consent was checked and reconfirmed along the data collection periods, in case they wanted to change their opinion and withdraw consent, which was not the case for any of them.

I provided my participants individually with transparent information regarding the content and the purpose of my study to assure their informed consent to my intrusion into and interference with their working life. At the same time, I encouraged them to contribute actively to the research by guiding, complementing and/or redirecting my steps to guarantee that their perspectives, standpoints, views and perceptions were valued and collected in the data as well as in the interpretation. I addressed the participants in their mother tongues, so that they would fully understand the significance and implications of their participation.

There were several aspects accompanied by pressure, in establishing proper contact with the participants that would lead to establish rapport or collecting proper, accurate, unaltered data. Nevertheless, I intended to be unbiased, approaching the participants with a completely open mind during all the stages of the research (O'Reilly, 2009). In terms of confidentiality, even if it is better to keep the real names as far as possible, with the permission of the participants, (O'Reilly, 2009), I only used the real name of Matti, the cabinetmaker who participated in the writing of Study III and whose prototype's construction I was able to document. I assigned pseudonyms to all the participants in the rest of the articles to avoid comparisons between their answers, situations, skills, or dispositions. However, I did not make changes to their personal details or backgrounds and used only necessary details that would not lead to identifying the participants.

I tried to respect all the possible ethical considerations that emerged during the research process, and, in particular, followed the guidelines on the ethical principles of research on humans and human behavior (TENK, 2019) and the guidelines for the responsible conduct of research (TENK, 2012). I believe that I respected the dignity of all the participants, without causing any risk or damage, and I followed in full measure the principles of integrity and accuracy in conducting this research (TENK, 2012). However, once Studies I, II and III were published, I had no control over how my text would be interpreted or understood by its readers (Pink, 2007). The work was performed in compliance with the standards set for conducting research and reporting the findings. Overall, the ethics of my research have been addressed in a practical manner by making adjustments to my procedures and behavior towards the participants. The adjustments were also made to how I handled the data and how the materials were obtained and used. As Dennis (2018) claims, as a researcher I cannot consider myself apart from the context of research. As an individual and as a researcher, I projected my image and my actions with respect and integrity towards those I was in contact with in order to build and maintain a trustworthy and professional relationship in terms of my research. As a researcher, I was there to learn their ways, understand their points of view and follow their practices. Simultaneously, I was also exposed to them, and as humans, even minimal contact often awakens feelings, assumptions, and impressions. As a researcher, it is my ethical duty to minimize as much as possible the impact and influence that my being provokes in the participants. On the other hand, a complete absence as a researcher would not be natural either. What makes sense is to aim to blend and fuse in the most natural way possible. With the ethnographic approach I did not try to generalize or make universal claims. The aim was to take a closer look at different individual cases. To accomplish this, a humble positioning, open and ready to receive was what I thought would work best. To position myself as a researcher was not entirely up to me, but also just as much a decision of the participants themselves. My responsibility was partial (Dennis, 2018). Position is a dimension relative to something else, and therefore the participants had a role in my positioning as a researcher. From my side, I knew I had to position myself. In their case, by meeting and agreeing to consent they validated me as a researcher. However, since the research process was lengthy, consent had to be renegotiated and my position as a researcher was ratified throughout the research.

As a researcher, not being an expert in the field of woodwork crafts or animal rearing, I had a significant lack of knowledge that was both an advantage and as a disadvantage. From an advantageous perspective, it allowed me to see the situations and settings with "fresh eyes" and curiosity. On the other hand, I considered it a part of my ethical responsibility to prepare myself as much as possible before trust was fully established. I read several manuals related to farming and animal husbandry. The participants themselves recommended the readings soon after I established contact (García & Gutiérrez, 1979; Manardi, 1980; Borghi, 1989; Portolano, 1990; Equipo 2100, 1991; Cañeque Martínez & Sancha Saldaña, 1998). The case with the cabinetmakers was similar. The cabinetmaker who became my gatekeeper provided me with two handbooks about cabinetmaking and wood techniques, tools and practices (Joyce, 1980; Newlands, 1992). By accepting those manuals to get a bit more acquainted with the field, I was showing them that I was ready to learn their ways, coming with a tabula rasa and open arms. The manuals proved to be extremely helpful and valuable as I prepared myself with basic knowledge and context elements, and they also became a tool for awareness and respect. The manuals helped me to deal with part of my vulnerability as a researcher, by providing elementary knowledge and facilitating merging into the settings. As a researcher I tried to understand and find my researcher-identity as a tool for connecting, understanding, and interacting with the participants. At the same time, the subjectivity of the researcher had to be partly separated from my self-identity. My researcher-identity was the one conceptualizing, working out the interrelations and depicting the connections seen in the context. My self-identity, outside that of the researcher, worked as a support and reinforcement, supplying doses of empathy, and developing close relationships with the participants. Both identities had to be aware of the existing cultural norms.

My fluency in the native language used with the participants, apart from increasing rapport, was of utmost significance influencing my behavior as a researcher (Kawulick, 2005). My language proficiency permitted the appropriate use of language and registers and facilitated interactions according to the existing sociocultural behaviors. In addition, language accuracy helped put aside possible preconceptions and confusions.

I positioned myself as a researcher with my five senses (Delamont, 2004). I listened as a researcher to the different voices of my participants. I observed as a researcher their movements, their actions and their body language. I smelled the barns and the workshops as a researcher. I tasted the dust and other signs of life that revealed a hidden story as a researcher. I touched, as a researcher, the different pieces of timber and felt the finishing smoothness or the roughness of the unfinished piece. My intuition and common sense were added to those senses to respect and reflect as a researcher.

5.5 Analysis

The process of inductive analysis had no clear starting point, but what was clear was the assumption that it is not possible to work without some mathematical knowledge (see Saló i Nevado & Pehkonen, 2018). My goal was to allow the theory to emerge from the data I gathered (O'Reilly, 2009). The analysis was an ongoing process of reflection while observing, interviewing, visiting the workshops and shadowing; as well as after the fieldwork concluded (Delamont, 2004). Being in the field, making decisions about access to the farm and to the workshops, the approach, visits, handling materials, and methods used were part of the analysis and its trustworthiness (Morse et al., 2002). Yet, given that Study I, II & III each occurred in a different context, the analysis steered toward a case-centered research (Stake, 1978) with three nuclei. The part of the analysis for Study I was based on the data collected on the farm through a more intensive data collection in terms of time spent in the field, compared to the data collection performed for Studies II and III. The second part of the analysis dealt with the data collected at the workshops for Study II, mostly through interviews and participant observation. The third part of the analysis focused on the shadowing of Matti and the process of building a prototype of the Pekki table. All three parts of the analysis were structurally similar, but the analysis as a whole was a multilayered interpretation of the farmers' and cabinetmakers' professional practices, their accounts of past events and reflections of the ongoing. Along with my observations and notes, these built up to a narrative (Cortazzi, 2001). The data was collected with the intention that it could help me portray the participants' view of what was going on, their stories. After being in the field, my first task was to transcribe the interviews into text files. For Study I, I transcribed the farmer's interviews a posteriori of the fieldwork. For Study II, the transcriptions from the cabinetmaker's interviews were performed after the visits by a third person who was a native Finnish speaker. Nevertheless, I revised them all and checked for possible misunderstandings as the transcriber had not been present when the interviews took place and there were some parts in English. Checking the transcribed interviews gave me an initial idea of the whole. For both Studies I and II, after the transcription, I read the text several times, looked for emerging themes, and coded the text for each of the participants' interviews (Thomas, 2006). I had expected to find some themes and not others. For Study I, I combined the data from the participant observation notes and the interviews and listed all the tasks identified on the farm, including their frequency of incidence and whether they were a routine activity for the farmers

(Atkinson & Coffey, 2001). I made a mind map of the activities and complemented it with details from the fieldnotes and the photographs. For Study II, after transcribing the interviews and reading the texts, I searched for emerging themes, which I linked to the participant observation notes from the field. In addition, I connected the photographs I had available to the different tasks described. I systematized smaller emerging topics into general preliminary themes and adjusted them according to the data. I also sorted through all the interviews of each study to further connect, strengthen and support the emerging themes and topics. Collaborative analysis was applied through the process by having a dialogue with the other authors of the study, to reach consensus about the interpretation of the data (Cornish et al., 2014) and to reassure rigor and consistency (Morse et al. 2002).

Most of the thematic grouping I did was task-related (for both farmers and cabinetmakers). Those themes were checked with the participant observation and shadowing fieldnotes. In Study III, I made a chronological list of the tasks, which I checked with Matti for incongruences or misinterpretations of the steps or the order of the tasks. I printed about 250 photographs and organized them all into a table in poster format. This was an important visual aid to conceive the process in Study III (i.e., the construction of the prototype) as a whole. I based my interpretations on what the data (collected by observation, active listening, and questioning) offered (Riessman, 2012). In Study III, my narrative attempted to reconstruct the story of what I saw and what was told by Matti (Cortazzi, 2001). Matti's input was extremely valuable. Throughout the analysis, I kept reading about different theoretical ideas with the intention to reach new perspectives to deal with the emerging themes, topics and information. I began with vernacular mathematics and problem solving and continued with creativity and design. My understanding came to light by interpreting the data that I had collected from the descriptions of the participants and the details captured from the different situations, the discussions and the interaction experienced with all the participants.

6 FINDINGS WITH-THE-GRAIN

As explained in Study II, in the world of cabinetmakers, when working with wood, 'with-the-grain' cuts are made in the wood parallel to the long axis to expose plain grain. Likewise, in this section 'Findings with-the-grain' is used as a metaphor suggesting that I will 'expose' and describe the findings of each of the studies, as if they were the grain of a piece of wood.

Study I, contextualized in a setting where the use of mathematics is not obvious (i.e., a farm), aimed to answer RQ1: *What mathematics do farmers use in their everyday work?* This study falls into the category of vernacular mathematics studies, as it was focused on a labor group's (i.e., farmers') use of mathematics (Eglash, 1997). The analysis of the data showed that most of the mathematical knowledge in use, and the farmers' behavior, focused on quantity, structure, space, and transformations along with problem solving. Overall, I found three different mathematical domains hidden in the activities of the farmers. The first was the measuring domain, as the farmers had to calculate different measures for volumes of feed and doses of medication for vaccinating the animals. For measuring, the farmers used different containers and pots; however, most of them were homemade and self-created, like the container to measure the milk replacement solution (see Picture 1). The second domain was the numerical and quantitative reasoning domain. Farmers understood and used numbers for different purposes, for example when identifying animals, counting fed calves or calculating the amount of feed ingredients to be mixed, as they prepared the feed themselves. Also, quantities were involved in the ratios to make corn silage or the exact medication to administer (see Picture 2). The third domain was geometrical-spatial reasoning. The farmers used different geometrical approaches when distributing and using the space in the barns for different tasks such as cleaning the barns, feeding or vaccinating. The farmers were clear examples of adults who reason intuitively, with common sense (see also Coben, 2000). They based their reasoning upon experiences within the specific context of the farm and they used a variety of methods to solve their problems. The responses of the farmers and the farmers' use of mathematics were shaped by the farm as a context, the tools and spaces available. Although the farmers' skills in formal mathematics might have been limited, and the level of mathematics they used in their daily life might have been somewhat unsophisticated, their basic numeracy allowed them to solve real-life, meaningful problems in complex context-specified situations (Evans, 2000).

Picture 1. (left) Feeding bottle for young calves and lambs, made of a beer bottle and a rubber teat. (right) Old kitchen pot used for measuring the 'milk feed'.

In addition to the mathematical domains, the data revealed that the farmers faced problem-solving situations where they had to use different strategies to find suitable solutions given the context. Study I presents two of those situations, both related to animal feeding and where the farmers use their numeracy skills. The first was about distributing the space in a barn for feeding calves and the second about using different objects as measuring tools for feeding the animals. In both situations, the context of the farm did not provide standard tools for solving the problems. The farmers had to use various strategies to find adequate solutions. Both situations were open-ended problems, or ill-structured, as Jonassen identifies them (1997). When optimizing the space in the barn for feeding the calves, Jaume the farmer, contemplated different alternatives and eliminated those that were not suitable for the purpose. He used the trial and error method, tested different models of fencing, divided the problem into sub-goals and, most importantly (according to him) kept it simple. All these steps were in line with the problem-solving steps described by Jonassen (2000) and Belland (2013). He obtained optimal results with fences that were made into simple geometric shapes. The mathematical requirements for finding a solution to the problem were not advanced, but the solution was vested between Jaume's numeracy skills, experience, and the resources available at the farm (Jonas, 2018). The data unveiled a second situation of problem solving at the farm where different objects were used for other purposes than the ones they were created for. In this way, the lack of conventional instruments for measuring forced Jaume and Elena to use other items as measuring tools and 'standardize' them as measurement units. This was the case with the milk measuring pot and the feeding buckets (see Picture 3).

Picture 2. (left) Scale and container for powdered milk to be mixed with water. (top right) Numbered calf. (bottom right) Ready corn silage.

Both items had a double identification. They were artifacts, as technical devices constructed or created according to specific goals (i.e., a pot to cook in and a bucket as a container for different substances) and as instruments, as having its own use modality, i.e., the pot as a measuring container and the bucket as a feeding device (Vérillon & Rabardel, 1995). The numeracy skills of the farmers were clearly present in this case and they were dependent on the context of the farm (Straesser, 2015). The farmers had a specific task to feed the animals and used an instrument for a well-defined purpose (see Figure 1 p. 24, where the key elements of Numeracy are represented).

Study II also falls into the category of vernacular mathematics studies but was focused on another labor group's use of mathematics, i.e., cabinetmakers (Eglash, 1997). In this case, the study aimed to answer three subsequent questions.

Picture 3. Feeding buckets.

For RQ2.1, *What is the mathematics used by cabinetmakers at work?* the analysis of the data revealed that the mathematics cabinetmakers identified and used in their work were in most cases very basic and therefore this finding was in line with many other previous studies about mathematics at work (e.g., Riall & Burghes, 2000; Coben, 2000; Hoyles, Noss & Pozzi, 2001; Williams & Wake, 2007).

Cabinetmakers used mathematics without self-evidently labelling it as mathematics (e.g., measurements and transformations) and only if they were able to use it (Coben, 2000). In the case of measurements, for instance, I was able to see how the cabinetmakers used measurements during the observations in the field, but this was not depicted by the interview data (see Picture 4). However, all cabinetmakers claimed that basic mathematical knowledge was sufficient in their daily work, as long as there was no situation that broke the daily routine. As basic mathematical knowledge, the cabinetmakers used basic operations (e.g., for measuring, cutting, assembling, etc.), estimations (e.g., calculating prices or amount of material), percentages and proportions (e.g., isometric), 3D geometry (e.g., areas, perimeters, volumes, etc.), measuring units, angles (especially when planning and making joints, adjusting blades, etc.) and basic trigonometry (for working with angles other than 90°). Not all the cabinetmakers had the same mathematical level and skills, but they used them to the fullest. This meant that where one of them was able to use trigonometry, and found it essential in woodcrafts, the other thought that it was possible to manage without trigonometry but admitted needing some other strategy to carry on.

Picture 4. Cabinetmaker measuring the distance from the blade of the band saw to the jig that will support the timber.

> Thomas: Quite seldom… sometimes we … we just had a case with ellipses…. we ended up with an equation of second or third degree. But very, very seldom and it is only just if you are interested in taking that kind of job. So, trigonometry is sufficient for cabinetmakers. But, of course also in trigonometry … it depends how you are involved in it. If you want to calculate angles of miter joints in various pyramids, you can get really hard equations. Then, involuntarily you will end up with equations of the second degree. When you have two variables, you cannot avoid it. But there are not many cabinetmakers who will bother their head with so difficult mathematics.

At the same time, their own mathematical skill level and knowledge seemed to restrict which projects they took in from clients. However, the data indicated that this restriction showed the level of engagement of the cabinetmaker and the contextualization (see Figure 2, p. 27), factors in line with the work from Straesser (2015), Wedege (2002) and Keogh et al. (2014). The data revealed as well that although mathematical knowledge and skills had a significant role in cabinetmakers' work and helped them to complete more demanding projects, they could never substitute for experience and skillful craftsmanship with the wood.

> Jacob: Wood is wood… and it is not always so precise. And if you just count, there remains a hole between the pieces, and you shouldn't let that happen… it is of a better quality if the pieces are together. If you consider that you are very good with trigonometry… you can use it very well, but for some reason it doesn't match. It's more important that the pieces are together.

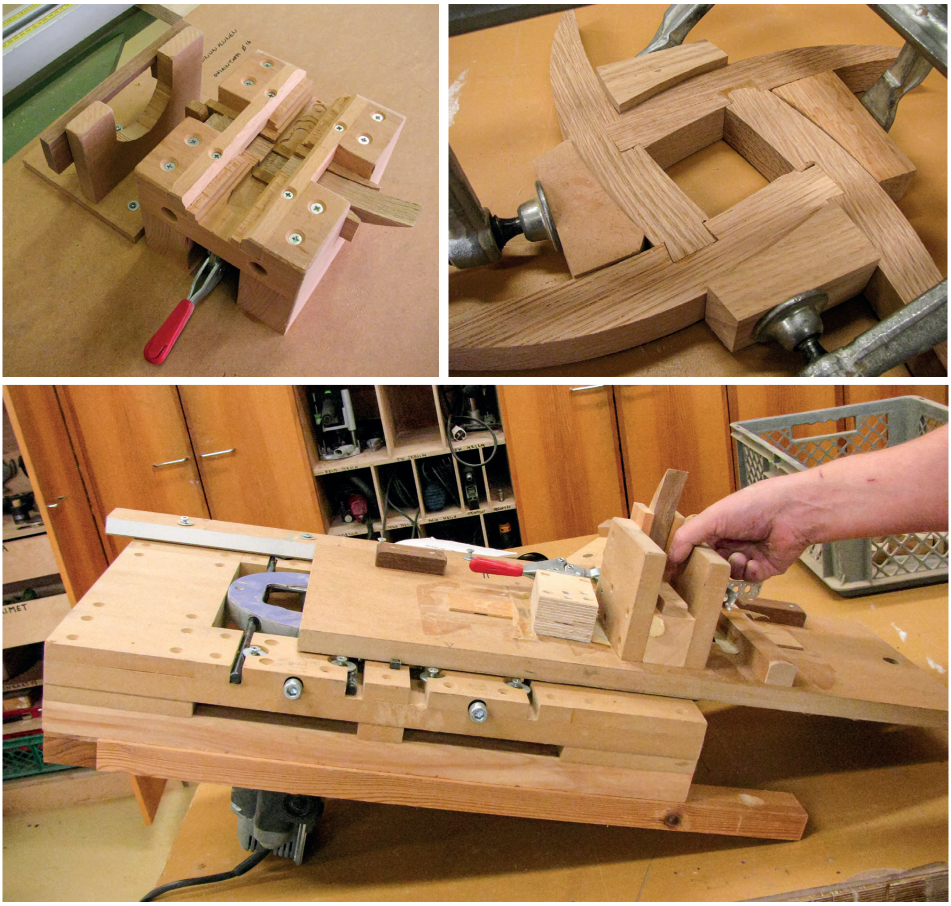
For RQ2.2, *What problem-solving situations do cabinetmakers face at work typically?* the analysis of the interviews showed that cabinetmakers faced different problem-solving situations in their daily tasks. As the following excerpt from the interviews reveals, their notion of problem solving matched the definition of problem solving by Schoenfeld (1983) and Tall (2013):

> Jacob: There is also very often that type of project where you are doing something for the first time, and you don't know (how to proceed).

I considered some problem-solving situations to be well-structured problems, like those in the following excerpt from Anthony. These situations were part of the routine job of a cabinetmaker, e.g., calculating costs and time approximations (Jonassen, 1997; FitzSimons & Boistrup, 2017).

> Anthony: Problem solving is an essential part of my work as a cabinetmaker. A great deal of my work tasks can be described as problem solving, starting with the customers' needs and ending with post-delivery issues. The most central problem in all designing and manufacturing is integrating outer appearance, functionality and costs.

Yet the data showed other types of problem-solving situations, mostly appearing when the cabinetmakers had to construct a jig (see different examples in Picture 5). These problem-solving situations (i.e., the jigs) were ill-structured, context-situated and open-ended, and the analysis of the data showed that mathematics was inevitably involved (Llorente, 1996; Jonassen, 1997). For instance, the jigs constructed for cutting and gluing together the arms of a trivet (see Picture 5) needed to allow the cuts to obtain a precise angle so the rest of the arms would match when the jig was used for gluing. Each jig could have been approached and constructed in different ways and different materials could have been used, leaving room for various solutions. This led to further pondering about the links between problem solving and creativity. The data also confirmed that a cabinetmaker's job typically included several problem-solving processes (e.g., building jigs). For RQ2.3, *How does the problem-solving process proceed?,* the analysis revealed that the stages of the problem-solving process and the creative process shared many similarities, yet, as the data reflected and according to Wimmer (2016), they should not be considered identical (see Figure 5). For both processes, the final goal is to find or to conceive a final product or solution. But the novelty may not rest only in the product, since it might also derive from the subject, the process, or the given setting (Amabile, 1983).



Picture 5. Examples of different jigs for building a trivet, specifically for shaping the arms (top left), assembling and gluing them (top right) and for cutting the right angle at the base of the arm (bottom).

However, when the novelty remains in the final product (e.g., the case of a dining table) the creative process becomes a design one. Sometimes this novelty in a design has a gradation and may be a mere improvement of some of the traits that define a previous product (see the different design procedures from Rosenman & Gero, 1993).

The analysis of the data showed that in the problem-solving process, what matters is always the viability of the solution (as in the case of jigs). Novelty is a condition of possibility in the creative process, as feasibility and practicality are for successful problem solving. Depending on its level of novelty and innovation, the solution attained by the problem-solving process may or may not be creative. When the cabinetmakers created a jig, their aim was to create something with a purpose whose value depended on its usefulness and not on its novelty (e.g., can the jig hold the piece of wood in the needed position and does it give it room for modifying a specific angle or not?).

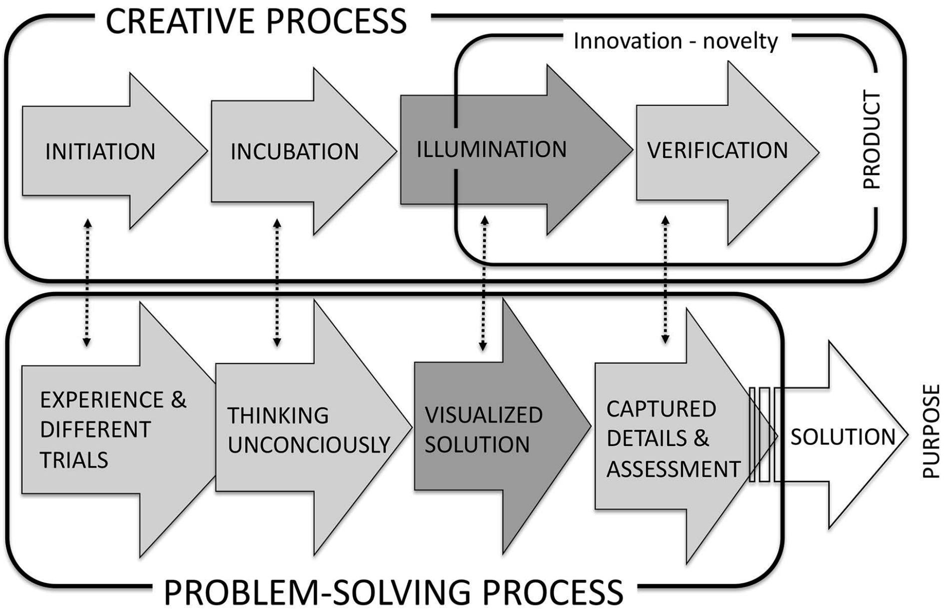


Figure 5. Problem-solving process and creative process based on the data from Study II.

Throughout the analysis it became very clear that the creation of a jig was a problem-solving situation, and it was not regarded by the cabinetmakers as a creative process, since the jig was meant to serve a definite purpose (see Figure 5). A problem-solving process may lead to a creative solution or to a less innovative one, but the validity of the solution does not depend on the level of creativity. On the contrary, a creative process without innovation nor novelty is not possible, as there is then no creativity.

Finally, the findings of Study III answered RQ3.1, *What is the cabinetmaker's process of designing and creating a table?* Through the data analysis I was able to make a table with all the steps of the design process of the Pekki table (see Table 3 and Picture 6). This table displays all the steps, including those not documented in situ, and the problem-solving situations that appeared during the process of construction of the prototype (i.e., emerging from professional practice, see Llorente, 1996). Those problem-solving situations determined the jigs that Matti had to create in order to proceed with the construction (see Saló i Nevado & Pehkonen, 2018).

PROCESS OF DESIGN & CONTRUCTION OF THE PEKKI TABLE			
1. Decision to design a table			
2. Looking for the right model, line and look			
3. Envisioned during a trip to Japan			
4. Construction of the prototype			Jigs
	PHASE I. Acquisition and timber seasoning		
	1	Cut the tree during winter	
	2	The log stays on the floor drying (1/2 – 1 year)	
	3	Horizontal sawing with the sawmill	
	4	Outdoor drying of the timber under a roof (1-2 years)	
	5	a) Pressure drying (two weeks) or b) Indoor drying (1-2 years)	
Documented	**PHASE II. Timber preparation, shooting and squaring edges for tabletop**		
	6	Trimming of the sides of the log, cut in half, diagonal splitting	*
	7	Plane the sides with a planer thicknesser	
	8	Measure the diagonals, cut exact pieces with the circular saw	
	9	Measure width and plane the wood	
	10	Measure thickness and plane the wood	*
	11	Measure length and sawmill the ends	
	PHASE III. Top of the table		
	12	Drill holes for the dowel joints	(*)
	13	Cut the dowels in octagonal shape	(*)
	14	Sand both ending of the dowels with sanding paper	(*)
	15	Glue the dowels to the board	(*)
	16	Put together two boards (x2)	*
	17	Put together two parts	
	18	Make holes (pins) for legs' dovetail joints: mill timber and drill a 7 degrees' angles sockets (dovetail rake)	*
	19	Polish the top of the table	
	PHASE IV. Legs of the table		
	20	Leg preparation steps 6-11 with different angle for diagonal split	*
	21	Leg pitching	*
	22	Cut the tails for the dovetail joints of the legs (rake)	*
	23	Cut the ending of the legs to be completely parallel to the floor	*
	24	Polish the legs	
	PHASE V. Finishing		
	25	Table name and number with a burning marker Pekki 1	
	26	Surface finishing and oiling 1	
	27	Hand polishing	
	28	Surface finishing	
	29	Ceremony and blessing of the table	

Table 3. Pekki table design and construction steps and stages. The shaded area indicates those steps and phases that I was able to shadow and observe directly. Jigs are marked with *. Those in brackets were not used in the prototype construction.

The analysis of the process reinforced the importance of using prototypes in the design of a product (see Aspelund & Kontzias, 2006; Dorta et al., 2008; Cross, 2011).

Once the problem-solving situations were located (i.e., a total of eight jigs involved in the prototype construction) I was able to go further into the analysis and answer RQ3.2, *How do the problem-solving situations influence the design and creation of a table and what is the role of the jigs within the design process?* The problem-solving situations that emerged during the creation of the Pekki table influenced the length, precision and progress of the process. The jigs created a drawback, were time-consuming, and interfered with the pace of the work. The perfect functionality of the jigs allowed precision in the cuts and the different parts of the table, and success translated into progress within the Pekki table process. However, the first thing revealed through the data was that the creativity of the problem-solving solution (i.e., the level of creativity of the jig) was not relevant at all. Instead, the success of the outcome of the problem-solving situation had an influence on the creative traits of the table and a direct impact on the design. All problem-solving situations consisted of a set of three smaller problems: how to proceed (how to saw, how to cut, how to glue…); what jig was needed to proceed within the construction process and how the jig was to be constructed. The analysis supported the understanding that Matti's jigs were, by definition, open-ended and ill-structured problems, as they were open to many different solutions (Becker & Shimada, 1997; Jonassen, 2000). At the same time, some of the jigs were examples of what Jonassen refers to as ill-structured problems that had become well-structured with practice (2000, p. 67). Those jigs had influence only on the construction process (i.e., jigs A, B, D and H, unshaded in the table, see Table 4).

To answer the last research question RQ3.3 *How are the processes of problem solving, design and creative process intertwined?* I put together the different processes in a table (see Table 4). Problem-solving processes seemed, based on the data and in this context, to mediate between the creative and design processes. All the jigs had an influence on the construction process at least, and their success allowed the construction to carry on. However, the data showed that, in addition to this, certain jigs allowed creativity to materialize and for the design to acquire new traits. The flexibility that Vidal referred to as a tool in the creative process to manage and adapt ideas (Vidal, 2009) is seen in our data obtained via problem solving. The jigs functioned as tools to make the ideas plausible. This was the mediating role, depicted in the data (see Figure 4 in section 2.4. Problem solving, p. 31).

PROBLEM-SOLVING SITUATIONS = JIGS	CREATIVE PROCESS	DESIGN PROCESS	CONSTRUCTION PROCESS
JIG A: assisting jig for trimming timber ends.	No special influence (NSI) in terms of creative process.	NSI	The cuts in any type of table. Measurement precision.
JIG B: assisting jig for cutting the timber in half to get two planks.	NSI	NSI	To proceed with the construction of the table. Measurement precision.
JIG C: assisting jig for diagonally splitting the planks for the top of the table.	Mattis's mental idea about the visual lines of the tabletop.	Clear influence on the design process. Added an original trait to the top of the table. The inclination of the bottom of the tabletop. Distinctive design.	To proceed with the construction of the table.
JIG D: assisting jig for holding the timber when using the planer thicknesser for measuring and planing the precise plank thickness.	NSI	NSI	To proceed with the construction of the table. Thickness precision.
JIG H: assembling jig for holding the top planks together until the glue is dry.	NSI	NSI	To proceed with the construction of the table.
JIG I: guiding jig for the router to obtain the exact shape on the bottom surface of the tabletop routing a 7-degree angle socket for the dovetail joints.	Matti's mental, innovative idea about the tightening system.	Influence on the design. To obtain the precise mortice shape, allowing the legs to be connected by friction. Innovative design trait.	To proceed with the construction of the table.
JIG J: subjecting and guiding jig for the manual leg pitching.	Matti's mental idea of the visual lines of the legs.	Influence on the design. To allow the visual effect that Matti wanted to obtain on the legs of the table.	To proceed with the construction of the table.
JIG K: guiding jig for the router to make a dovetail in each leg. Complementary jig to jig I.	Matti's mental idea of the tightening system.	Influence on the design. To obtain a precise dovetail shape for the leg mortice to fasten the legs by friction. Innovative design trait. Detachable legs.	To proceed with the construction of the table.

Table 4. Jigs and their influence in the creative, design and construction processes (shaded areas). Simplified table from Saló i Nevado et al. (2020).

Moreover, the analysis of our data indicated that the design process seemed to take its position between the illumination and verification phases of Wallas' creative process (Wallas, 1926; Hadamard, 1945), when the materialization of the product is unavoidable.

For Matti, craftsman and designer, the materialization is the prototype construction (Risatti, 2007; Temeltas, 2017), which has a significant role in the design process as well as in the creative process. In line with previous research, the data indicated that the prototype made it possible for Matti to evaluate whether his ideas and vision regarding the table worked or not (see Goel, 1995; Aspelund & Kontzias, 2006; Brown, 2008; Cross, 2011). Regarding the creative process, the data confirmed that, at times, it had an impact on the design process (due to the level of creativity used). The same seemed to have happened during the problem-solving processes as the solution may or may not have been creative. In the case of jigs, the creativity was irrelevant for the purpose. This translated into the conclusion that creativity is not a *sine qua non* of problem solving (Saló i Nevado & Pehkonen, 2018).

Picture 6. The Pekki table. Photograph by Jonna Öhrnberg.

7 DISCUSSION

The 'across-the -grain' cuts are made across the face of a board, revealing its end grain. In the sections that follow, I pull different themes for discussion from across the three studies.

7.1 Numeracy, Mathematical Knowledge in Use and Problem Solving Across-the-Grain

All three studies actively deal with human participants within their work context. The link between a person and their working context is their numeracy, since, to make sense of the surroundings, one needs to use numeracy skills (Evans, 2000). Throughout all three studies, different elements indicated that the numeracy of the participants, both farmers and cabinetmakers, was activated and in use. For instance, the mathematical knowledge in use helped the participants to become situated and aware of what was going on (e.g., time, location, direction, tasks, etc.) and the purpose and the tasks had to be adjusted to the context (as shown in Figure 1, p. 24). Even if, in the theoretical section of this thesis, I positioned problem solving as a part of the skills, competences and abilities within numeracy, the data of this research did not show problem solving as an individual item, but as a phenomenon embracing and having a broader impact on other elements of numeracy. Furthermore, the data showed that problem solving in the case of both farmers and cabinetmakers was not possible without mathematical knowledge as well as other components of numeracy. The farmers needed all the elements that the farm provided as a context to succeed in their daily tasks. These elements were, for example, materials, spaces, or tools. At the same time, the analysis pointed out that success in the problem-solving situations (both at the farm or at the workshops) made sense only within the given settings, specifically in the case of the jigs. The data also indicated that the role and impact of the mathematical knowledge in use is restricted but necessary to a certain extent. For the cabinetmakers, basic mathematical knowledge was needed in their routines, as well as in other tasks (FitzSimons & Boistrup, 2017); even if they only associated the mathematics in use with those situations where numbers were explicit (Wedege, 2002; Keogh et al., 2018). Yet, as depicted from the interviews, they considered that advanced mathematical knowledge was not imperative. For them, the context provided alternatives that, combined with their numeracy skills, responded to the needs of the situation. The data supports the link between numeracy and problem solving and shows that they are both dependent on mathematical knowledge in use (especially in the case of the cabinetmakers) as it is embedded in their daily tasks and routines in one way or another. However, as seen in the case of the cabinetmakers, the mathematical knowledge in use during problem-solving situations is selected by the individuals, depending on the situation they are dealing with. In Figure 6, I illustrate the idea that different contexts provide different problem-solving situations (PS) and that the numeracy of the subject puts in use certain mathematical knowledge, therefore only the parts required. Different problem-solving situations request different mathematical knowledge in use, even if they belong in the same context. Every person copes differently according to their own skills, experience and knowledge. Figure 6 could be applied to each cabinetmaker and farmer and the resulting figures would be exclusive for each person.

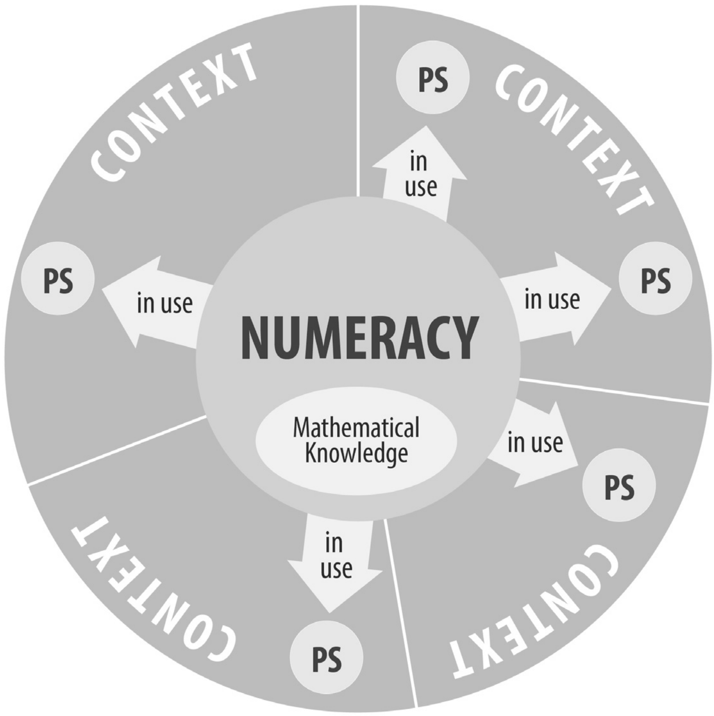


Figure 6. Example of how the mathematical knowledge is used in different contexts to deal with different problem-solving situations (PS). Only the necessary part of mathematical knowledge is utilized (in use).

The context sections in Figure 6 represent all the factors that Keogh et al. (2014; 2018) described and that influence the workplace. The data of this research indicated, in line with Keogh at al., that the link between numeracy, workplace and problem solving is shaped by a mixture of different factors, apart from the mathematical knowledge of the subject. For example, the farmers and cabinetmakers were tied to the purpose of each of their tasks, whether it was feeding animals, cutting timber, or measuring planks. As described in the interviews, they chose various strategies accordingly. The following excerpt from Thomas' interview exemplifies what Keogh et al. referred to as accountability. It also included decision-making connected to experience and common sense.

> Thomas: … and hand in hand with the math problem, one has to always add decisions. I have always concentrated on decisions. Decisions, decisions, decisions. If you make the wrong decisions, then you can change it or make it again. But if you are not making decisions…

Interestingly, according to our data and within the given settings, it became evident that those factors contextualizing the workplace were also contextualizing the given problem-solving situations (e.g., limitations due to materials, time restrictions, risk taking or decision making among others; see Keogh et al. 2014; 2018) and, at the same time, the mathematical knowledge in use. The mathematical knowledge used by the farmers and the cabinetmakers ixn our study is, after all, the vernacular mathematics of these two labor groups (Eglash, 1997). The context in which this knowledge is used contains problem-solving situations that, if successful, might become routine tasks, as solved situations cannot be identified as problem solving once the gap is bridged. In our data, this was the case of those jigs that the cabinetmakers just built from experience, as they had had to construct similar ones in previous projects (Jonassen, 2000). Problem solving allows mathematical knowledge in use to evolve and develop. It provokes a renegotiation at an individual level (Van Oers, 2001) and, sometimes, in a more generalized form by the hand of technology, for example, might force the mathematical knowledge to be modified and new skills acquired.

7.2 Problem Solving, Design and Creative Process Across-the-Grain

Examining the problem solving and the mathematical knowledge used by the farmers and, especially, by the cabinetmakers at work, steered the research towards the creative process. It was not an unexpected turn as, in the work of Keogh et al. (2014; 2018) for example, the level of creativity seems to be a relevant factor in the workplace (e.g., in the creativity of the outcome or during the development of a task at work). My analysis supported the idea that creativity within problem-solving situations seems to be irrelevant and only gains significance within the creative or design processes.

However, the creative process was seen not only in the problem-solving situations, but also emerged from an unexpected place. During the course of Study II, when I realized how significant it could be to document a cabinetmaker's job from the beginning to the end, I had in mind a regular client order, such as a kitchen cabinet, a chair or a wardrobe. When Matti expressed his intention to create something new, I simply thought of it as a project or a job, not yet realizing the fact that this project, per se, was a creative project. The fact that Matti chose a table as the resulting product, converted it into a design project (Vidal, 2009). Matti's idea was to create a new design to be offered to different customers. These circumstances brought me to reconsider the position of the cabinetmakers as wood craftsmen, since, in this case, Matti also qualified as a designer (Risatti, 2007; Sennett, 2008). Further implications arose as well, as the whole job (process) was a creative process and design process at the same time. According to the design procedures described by Rosenman and Gero (1993), the case of the Pekki table is a design by mutation, as it is a table by definition, but Matti modified some of the specific features. Those features were what the analysis showed as distinctive traits of the Pekki table: slanted bottom surface of the top of the table, absence of nails, screws or glue in the joints and a friction tightening system.

Picture 7. Matti making a diagonal split cut with the band saw with the help of a jig.

At the same time, two of these traits had an important mathematical component related to angles and measurements that represented a problem-solving situation. Matti required the aid of different jigs to proceed. First, the slanted bottom of the surface of the top of the table was possible to obtain thanks to a jig for diagonally splitting the wood at the right angle. The friction tightening system was also made possible by using two jigs that permitted the router to cut at the right angle in the dovetail joints (i.e., the dovetail rakes for both the pins and for the tails). Accordingly, both design traits were possible because of the success of the jigs (i.e., problem-solving situations).

In the case of Study I, the farmers were creative in the solutions that they found for the problem-solving situations that emerged at the farm. The data revealed that Jaume and Elena used their creativity during the problem-solving situations explored (i.e., distributing spaces and finding feeding utensils as measuring tools). Once more, the mathematical component was evident (i.e., measuring and spatial reasoning) but the creativity in their solutions was not relevant. In other words, the utensils used as measuring tools were unique and could be regarded as creative solutions, yet there was no value attributed to their innovation. Instead, value was awarded based on their usefulness and purpose.

8 CONCLUDING NOTES

This research was intended to examine problem solving and the mathematical knowledge used at work through the working context of two farmers and four cabinetmakers. More specifically the dissertation had as its objectives to examine different work situations in a farm where the use of mathematics was not obvious; to find out the mathematics at use in a cabinetmaker's workshop as well as the emerging problem-solving situations; and to investigate the connection between problem-solving processes, creative processes and design woodworking processes. The findings suggest that in both settings (i.e., the farm and the workshops) basic mathematical knowledge is in use and problem-solving situations have a clear mathematical component. Farmers mostly used elements from the measuring domain, from the numerical and quantitative reasoning domain, and from the geometrical-spatial reasoning domain. On the other hand, the findings indicated that cabinetmakers used basic mathematical knowledge. These findings were broadly in line with those of researchers such as Hoyles et al. (2001) and Williams and Wake (2007). In addition, I found that the context shaped many elements, such as the strategies used by the participants, the tools available or the factors that influenced the tasks and the problem solving that emerged. The context was present in numeracy as well as in routines and problem solving. At the same time, the context limited the creativity and design processes. These findings were also consistent with previous research from Keogh et al. (2014; 2018) and Wedege (2002; 2007). Moreover, problem-solving situations were found to emerge from both settings, and in the case of the cabinetmakers, the situations appeared mostly when the cabinetmakers had to construct a jig for proceeding with a job. The data indicated that jigs were ill-structured, open-ended and with a solid mathematical component (Llorente, 1996; Jonassen, 1997). In addition, the problem-solving situations seemed to be the catalyst for success in the creative process and design of a table examined in the study. However, it should be borne in mind that the findings of this study are restricted to the given contexts and participants (farmers and cabinetmakers). Further research should focus on the implications of the combination of efficient problem-solving skills together with more than basic mathematical knowledge used in different work contexts. In line with FitzSimons' work (2013; 2019), I argue the importance of studies such as this one, attempting to develop an understanding of problem-solving features and the contextual mathematical knowledge in use in an era of rapid changes and technological development (Tall, 2013). From the point of view of work and education, the main question derived from this work is what would be considered valuable and useful mathematical knowledge for the cabinetmakers or the farmers and what professional mathematical knowledge and skills would be worth pursuing (Van Oers, 2001).

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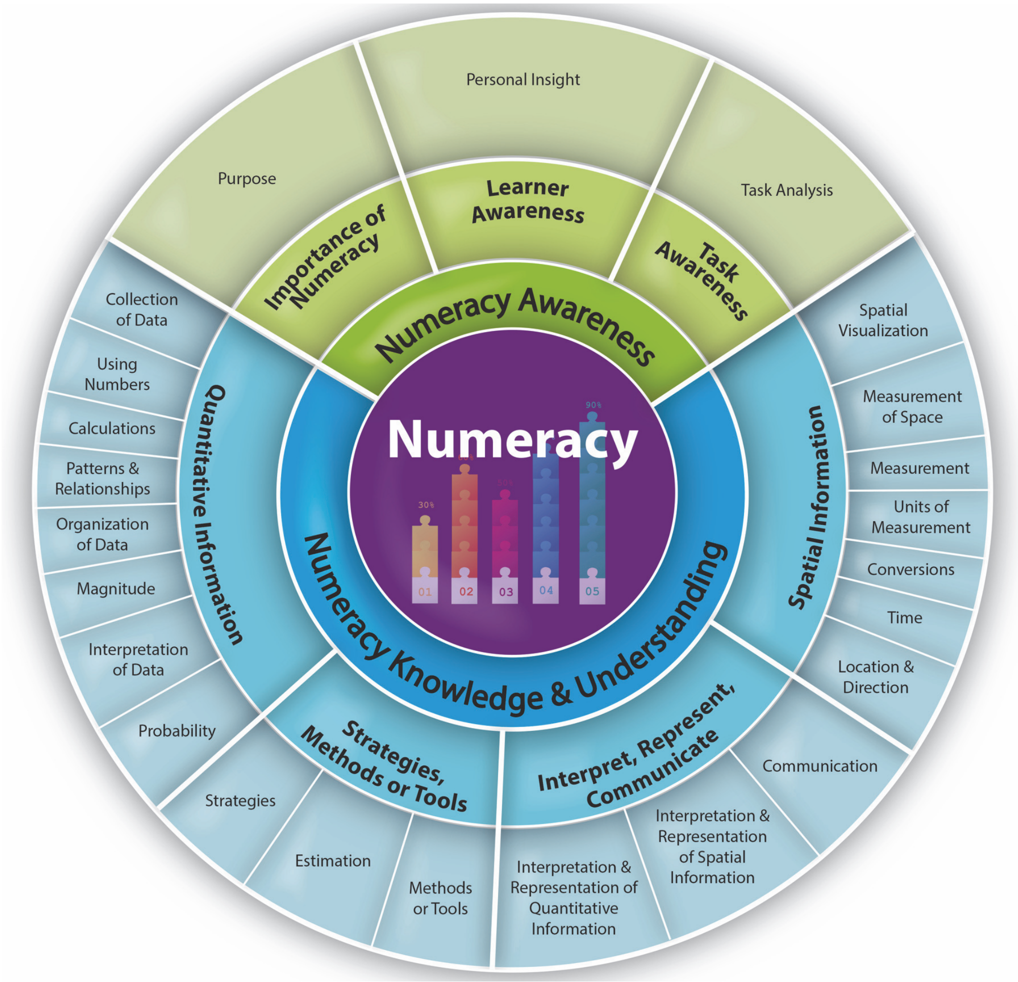
10 APPENDICES

Appendix A: Numeracy Progressions



Alberta Education, 2019:
Retrieved from:
https://arpdcresources.ca/consortia/literacy-numeracy-progressions/

In:
https://education.alberta.ca/literacy-and-numeracy/about-literacy-and-numeracy/

Appendix B: Amabile's creative process model

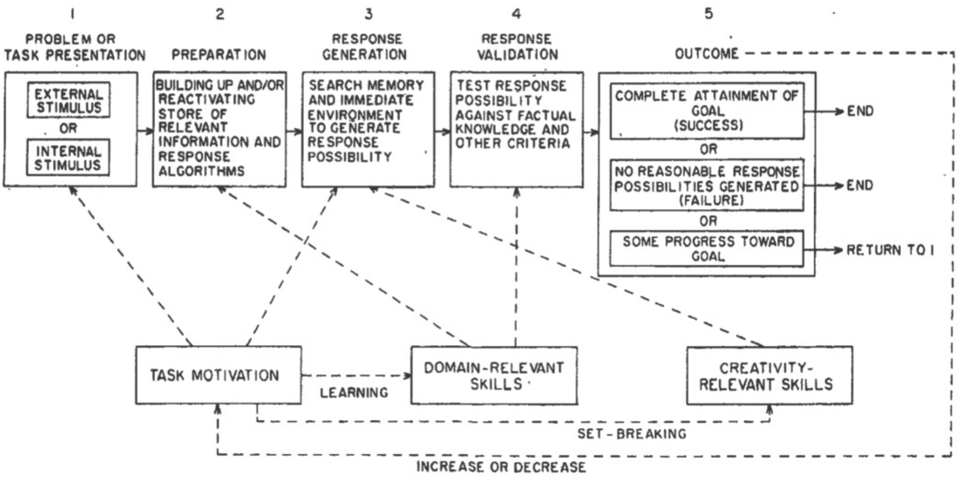


Figure 2. **Componential framework of creativity. (Broken lines indicate the influence of particular factors on others. Solid lines indicate the sequence of steps in the process. Only direct and primary influences are depicted here.)**

From page 367, in Amabile, T. M. (1983). The social psychology of creativity. *Journal of Personality and Social Psychology*, 45(2), 357-376.

Appendix C

Guiding interview questions for farmers and cabinetmakers:

- How long have you worked at a farm/been a cabinetmaker?
- Could you describe a typical day at work? What do you do?
- What do you do when you get here?
- Can you describe a typical task at work?
- What type of tools do you mostly use or need?
- What about non-traditional tools?
- What do you do when you get a client contacting you?
- Please walk me through each and every step.
- What do you do when you deal with…?
- Could you show me? Could you tell me what you just did?
- Could you give me some examples?
- Can you put in words what you just did to…?
- What are the mathematics that are clearly involved in this task?
- How do you calculate that?
- When do you use this? How do you use this?

Questions for Cabinetmakers:

- Do you recall something on the job that was not taught at school?
- Could you describe a typical day at work? Could you describe typical tasks at work?
- What are the mathematics that are clearly involved in the tasks of a normal day? How do you use mathematics?
- What are the typical /usual problems you must solve? What are the typical situations where you face problems?
- Do you recall something on-the-job that was not taught at school?
- Describe what tools are mostly used. Non-traditional tools.
- What type of maths do you use at work that was not taught at school? How is maths at work different from the one they taught at school? Do you use mathematics at work? How? What type? Could you show me?
- Questions about a concrete task and how it is carried out. [All the informants could describe and be asked about the same task.]
- Use your own terms to describe…
- What are the typical tasks where mathematics is clearly involved?
- Are there different levels of mathematics in use? If so, describe them.
- When was the last time you experienced having a problem-solving situation, where you wished you had more mathematical knowledge/tools available?

Appendix D

Examples of photographs taken at the farm:

Picture 8. Manual milk feeding mechanism on rails for calves.

Picture 9. Grain distributor for feeding purposes.

Picture 10. Livestock at one of the feeding stations.

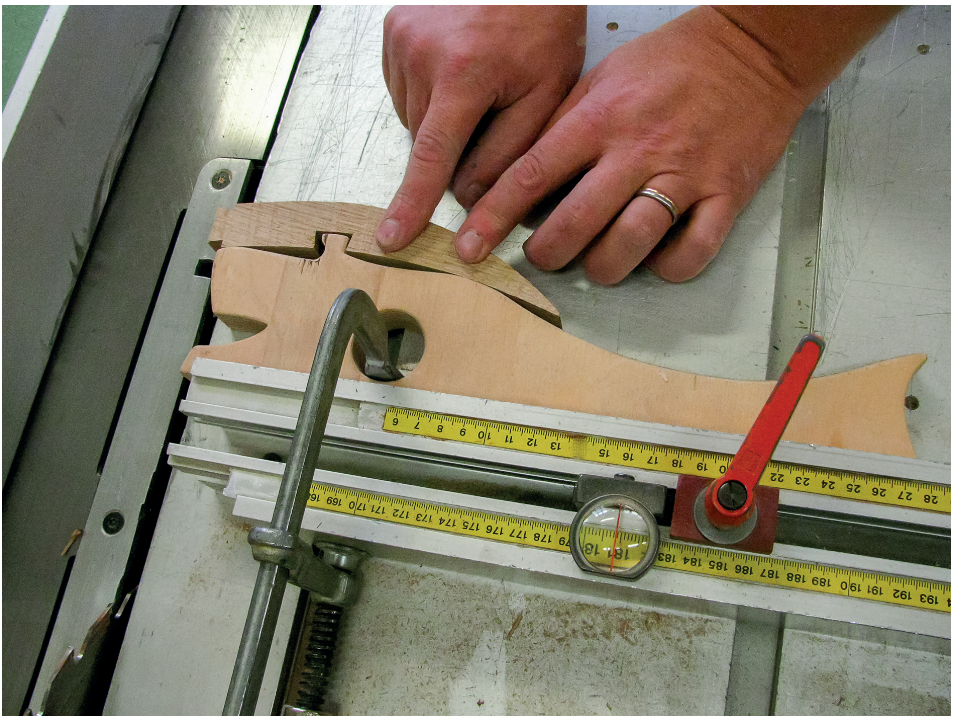
Picture 11. Barn.

Picture 12. The youngest calves inside the barn showing the fencing created for division purposes.

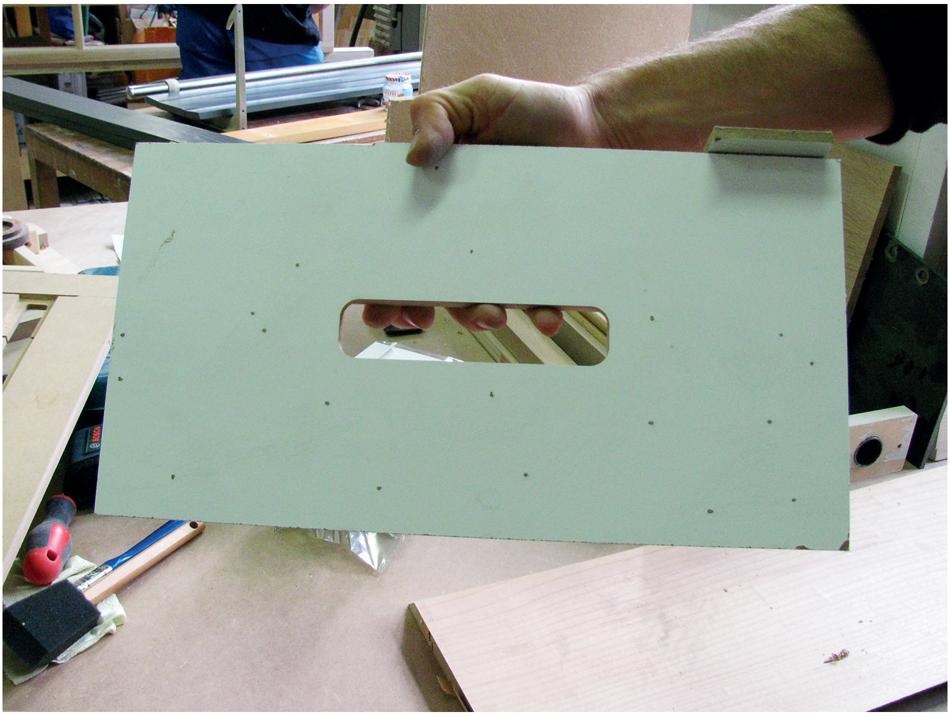
Picture 13. Fencing used for the vaccination of the calves.

Picture 14. Using a tractor to move the metal fences to cordon off spaces.

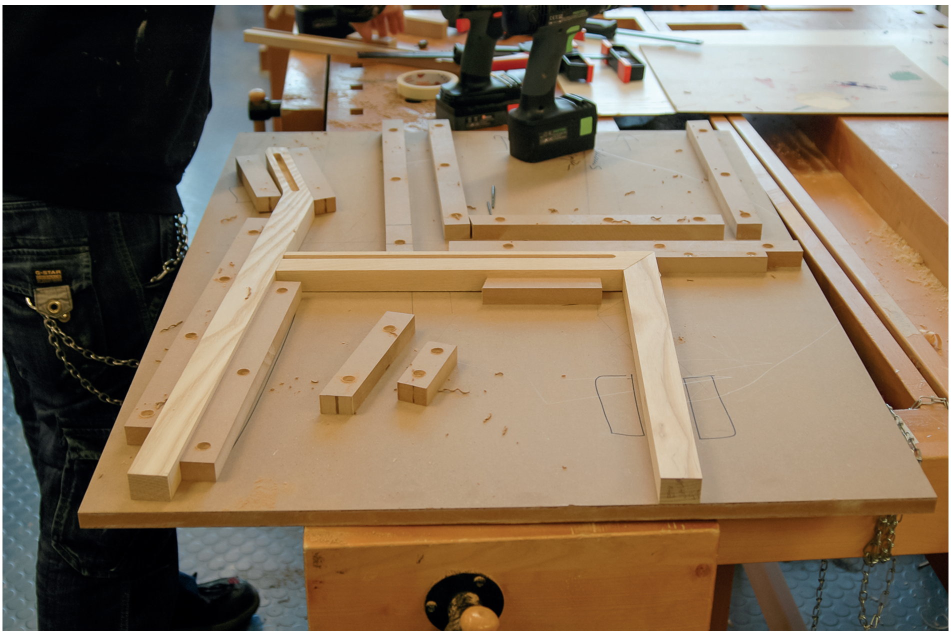
Examples of photographs taken at the workshops:



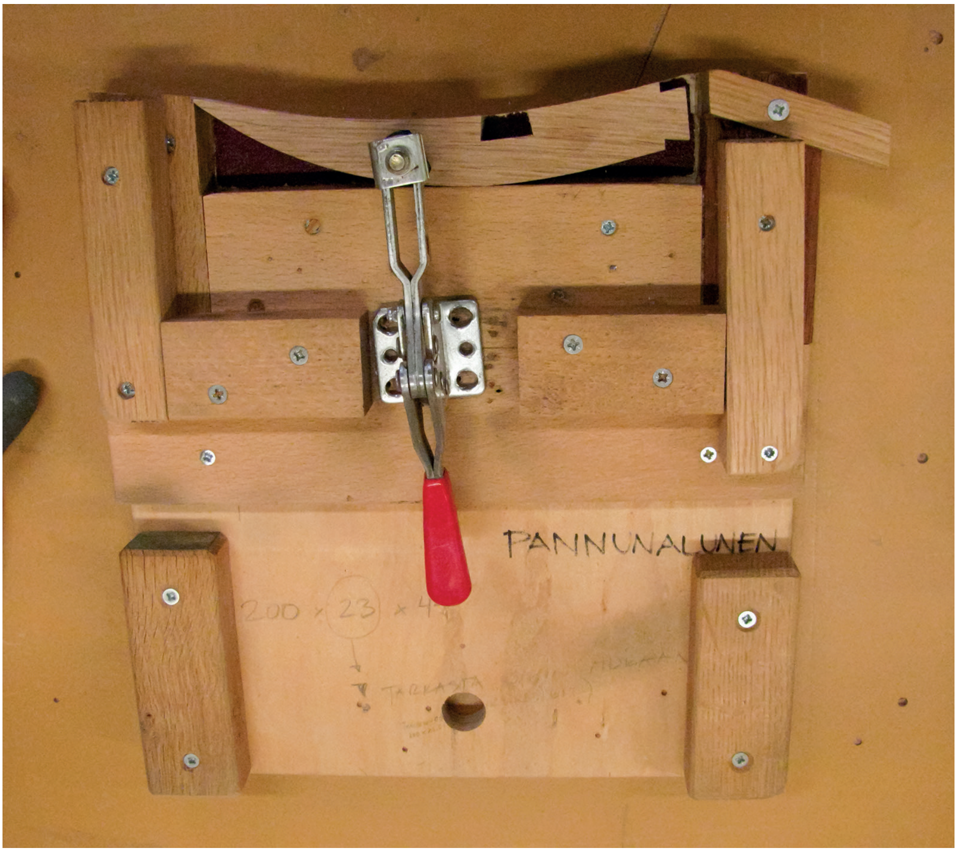
Picture 15. Jig to hold the arm of a trivet in the right position for the cutting of its joint tail.



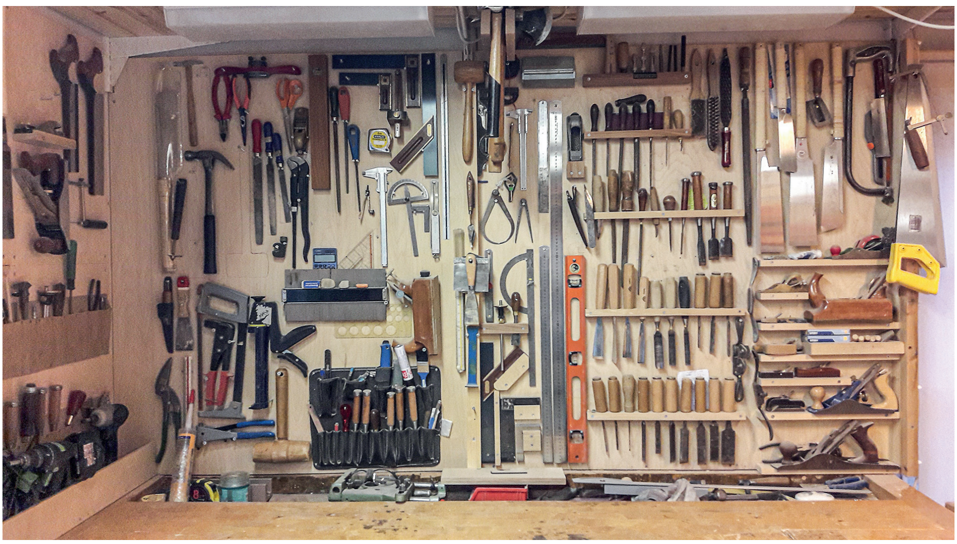
Picture 16. Guiding jig for the router.



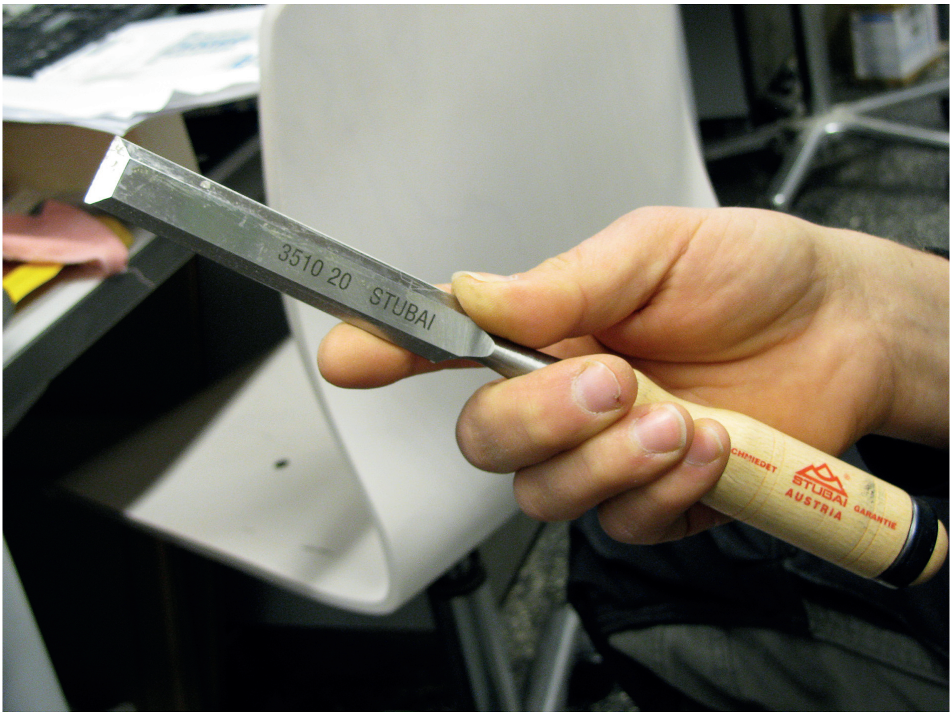
Picture 17. Assembling jig to hold a chair.



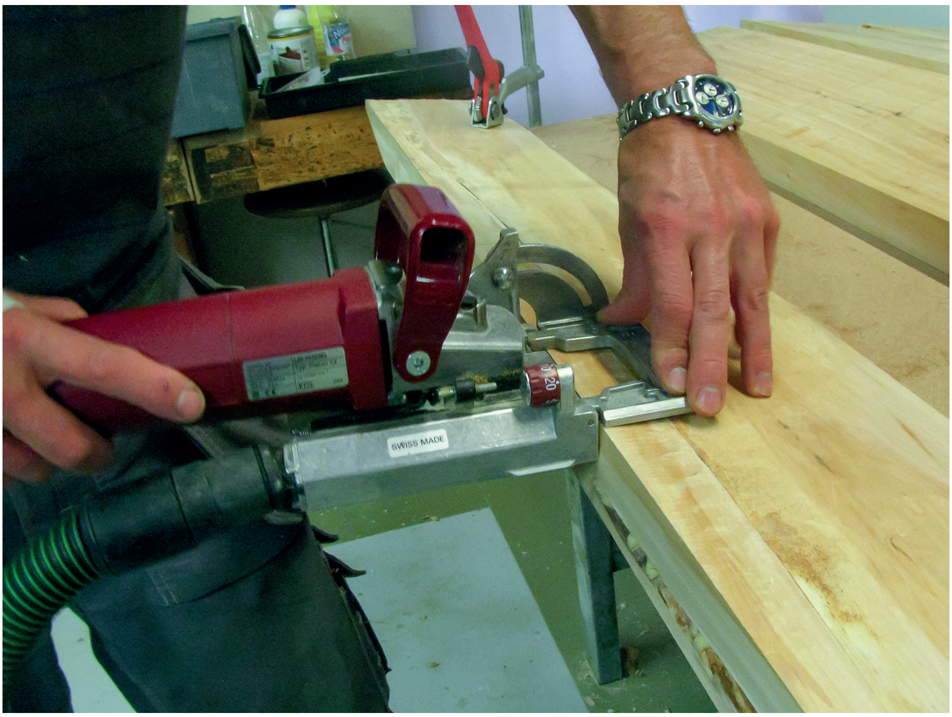
Picture 18. Support jig for a trivet arm.



Picture 19. Tools in Matti's workshop.



Picture 20. Chisel.



Picture 21. Dowel jointer.

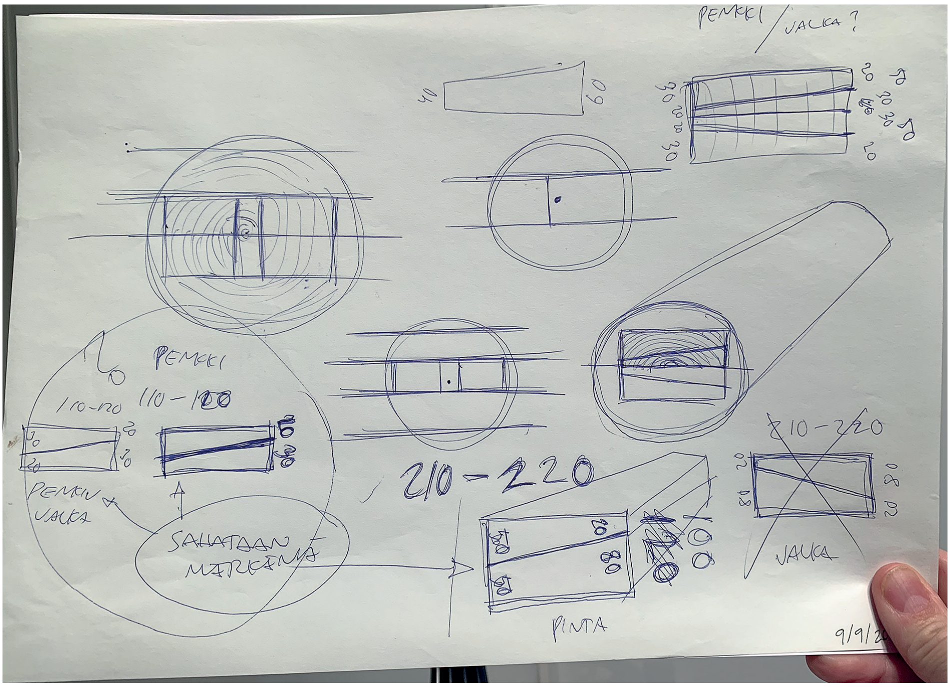
Picture 22. Matti making a jig to support the bandsaw to make a diagonal cut.

Picture 23. Matti using the bandsaw to make a diagonal split.

Picture 24. Matti contemplating.

Picture 25. Matti preparing the components for the tabletop.

Picture 26. A jig to cut the legs so that the table is balanced.



Picture 27. Matti's sketches of the timber cut for the Pekki table.

11 ORIGINAL ARTICLES

Farmers do use mathematics: the case of animal feeding

LAIA SALÓ I NEVADO, GUNILLA HOLM AND LEILA PEHKONEN

This article presents findings from a study on the use of mathematics in the context of a farm. Ethnographic methods were used for the data collection and ethnomathematics provides the theoretical framework guiding the analysis. We present two different situations, as examples of ethnomathematics, in which the farmers make use of mathematics in daily life situations on a farm. The first situation has to do with how one of the farmers dealt with a barn as a space for feeding calves. The second situation is about the use of different objects as measuring tools.

The abundant presence of mathematics in daily life and the fact that many people do learn and use mathematics outside school and beyond the formal usages are strong points for teaching mathematics using out-of-school experiences. However, in order to do so, it is useful to identify the use of mathematical knowledge outside the formal school environment of mathematics. Mathematics knowledge is included everywhere in our everyday settings (Nunes, 1992). In this article, mathematics is regarded as the knowledge and behaviour embedded in dealing with change, structure, space and transformation complemented with what Van Oers describes as "the observance of particular rules, the use of particular concepts and tools, the engagement with certain values," (Van Oers, 2002, p. 72). Daily life and tasks at workplaces require mathematical knowledge, sometimes more as routines, sometimes in a more problem-oriented way. This is what FitzSimons refers to as adult numeracy, since it implies "a practical aspect to using mathematical ideas and techniques, whether in the paid workforce or in unpaid family and community situations" (2008, pp. 9–10). According to FitzSimons, numeracy "relies on

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Saló i Nevado, L., Holm, G. & Pehkonen, L. (2011). Farmers do use mathematics: the case of animal feeding. *Nordic Studies in Mathematics Education*, 16 (3), 43–63.

common sense, and it is context-specific and context-dependent, directed towards the achievement of specific, immediate, and highly relevant goals" (2008, p. 9). As will be seen in the following section, mathematics and mathematical activities in workplaces at all levels have interested many researchers. However, we know very little about how mathematics is used at the level of farms. Most of the studies are directed towards the formulas used in a farm for economical efficiency and farm management (see Glydon, 2005; Mitchell, 2003).

Activities, such as feeding, shearing, vaccinating, preparing the animals for reproduction, or simply identifying animals, are tasks from the farmers' daily routine that may include various mathematical elements. A farm is a good example of an informal milieu in which various uses of mathematics may be present. Adults on a farm cope with the sophisticated demands of the daily practices involving many mathematical elements. Often, these practices are forgotten in the classroom, and there is no link between the reality of daily life and the formal mathematics taught in the classroom. The recognition of adult competency in mathematics, when dealing with daily practices in a rural environment, can be a strong and persuasive tool to use in adult education programmes (Tusting & Barton, 2003, FitzSimons & Wedege, 2007).

This article presents findings from a study showing that farmers use mathematics in daily activities on a cattle farm. We are interested in knowing what mathematics the farmers use in certain situations. The purpose of this article is not to make pedagogical implications derived from the use of mathematics in a farm; but to investigate a natural setting where the use of mathematics is not obvious and to emphasize the process the farmers are involved in. In this paper we analyze two different situations in which farmers make use of mathematics.

Theoretical framework

The idea of taking into account the mathematical content and procedures outside the formal environment of mathematics is not new; many others have insisted on the importance of such considerations (e.g. Angulo, 1994; Cabello, 1997; Ascher, 2004). Before people attend formal schooling, many mathematical concepts are already acquired through informal interaction with other members of the society. Butterworth (1999) refers to this kind of learning as a socio-cultural phenomenon. Accordingly, the context becomes the key for understanding the use of mathematics in everyday life (Gainsburg, 2005). Understanding the bonds between context and mathematics is the major focus of ethnomathematical enquiry; ethnomathematics being the theoretical approach which provides the foundation for this study.

Accorging to Ron Eglash (1997, p. 79) ”ethnomathematics is typically defined as the study of mathematical concepts in small-scale or indigenous cultures”. The concept of ethnomathematics was coined in 1977 by D’Ambrosio during a presentation for the American Association for the Advancement of Science. Over the years, the concept has been defined and developed by many researchers (for example Barton, 1996; Pompeu, 1994; Ascher, 2004 or Knijnik, 2003). Different lines of investigation have been followed. For example, Knijnik (1995) presented a more socio-political view of ethnomathematics related to the landless people in Brasil. Oliveras (1996) carried out a study about the mathematics identified in Spanish crafts emphasizing the ethno-didactics involved in the process of production. Clareto (2003) researched the space perception of school children in a small fishing community in Brasil.

D’Ambrosio defined ethnomathematics as ”the mathematics which is practised among identifiable cultural groups, such as national-tribal societies, labour groups, children of a certain age bracket, professional classes, and so on” (D’Ambrosio, 1985, p.45). This definition indicates that ethnomathematics is not necessarily about new mathematical knowledge. On the contrary ethnomathematics is foremost about the use of already known mathematical knowledge. Therefore, D’Ambrosio insists on describing ethnomathematics as a ”research program in the history and philosophy of mathematics, with pedagogical implications, focusing the arts techniques [tics] of explaining, understanding and coping with [mathema] different socio-cultural environments [ethno]” (D’Ambrosio, 2006, IX).

Ethnomathematics can been seen as an intersection set between cultural anthropology, formal (institutional) mathematics and mathematical modelling. Ethnomathematics utilizes mathematical modelling to solve real-life problems, and translates them into modern mathematical language system (see Orey & Rosa, 2006). Eglash (1997) locates ethnomathematics as one of the five subfields in the anthropology of mathematics with emphasis on small-scale indigenous or traditional societies.

Through the lenses of ethnomathematics, it is easier to understand the cultural dynamics within which knowledge is created. However, D’Ambrosio argues that ethnomathematics is not a folkloristic view of how other cultures or cultural groups do counting, measuring or distributing. It is not the study of rare phenomena or curiosities (D’Ambrosio, 1999). On the contrary, ethnomathematics focuses on ”how the knowledge, specifically mathematical knowledge, is generated, intellectually and socially organized and diffused” (D’Ambrosio, 1999, p.52). Therefore, as a research field, it has valuable pedagogical implications.

Jama Musse Jama claims (1999) that often examples of the local culture may be used for introducing mathematical arguments. Ethnomathematics allows a good response to the problems regarding the cultural component of education. In other words, ethnomathematics takes a step further towards documenting and including social practices and procedures regarding mathematics into the formal education.

In the last decades, mathematics educators have made many efforts to include ethnomathematical ideas in the formal, institutional mathematics learning and curriculum texts. Some have seen ethnomathematics as the missing step to be added and to complete mathematics education within the framework of diversity (see Presmeg, 1998; Dickenson-Jones, 2008). One example is Luitel's and Taylor's (2007) attempt to create a culturally contextualized model of mathematics education in Nepal. The model held ethnomathematics as a theoretical reference that help students to develop their cultural capital, and the use of mathematics becomes more beneficial for learners.

Situated cognition offers another perspective of a culture's influence on mathematics learning. This is the line followed by those in ethnomathematics that study the use of mathematics by adults and students in daily life situations as opposed to the formal mathematics of school. Barton (1996) illustrated good examples of this type of research like the analysis of Saxe (1988) of Brazilian candy sellers and Carraher, Carraher and Schliemann (1985) of illiterate people in Brasil, or Lancy's (1983) work on the Kewa's counting system and calendar in Papua New Guinea.

In this article, the ethnomathematics refers to the practice and uses of mathematics in a specific context. Our understanding about ethnomathematics in this study is very near to Eglash's (1997) concept of vernacular mathematics, which he sees more or less separate from ethnomathematics, but nevertheless as a subfield of the anthropology of mathematics. Eglash refers by vernacular mathematics to the use of mathematics of those who are distinctly outside any mathematical professionalism (of either west or non-west) and would not qualify under the anthropological category of an "ancient cultural tradition". This kind of mathematics could also be called folk mathematics – mathematics that folks do (Maier, 1980).

The context that frames this study is a calf-rearing farm. Most of the time, the context modifies or even transforms the use of mathematics. From an ethnomathematical perspective, the study of the use and the practice of mathematics in a rural setting such as a farm, may lead to a better understanding and a more effective development of adult mathematics education, since ethnomathematics engages the personal experiences with the use of mathematics in everyday life. The everyday life is what Vithal and Skovsmose (1997) label as everyday settings.

Interestingly, the use of mathematics when carrying out daily routines is not always a conscious act (Wedege, 2002). However, when mathematics is purposely contextualized, it becomes more culturally relevant and thereby makes a difference to the learner and that consciousness may emerge.

Methodology

Barton (1996) defined four types of empirical methodologies that differentiate ethnomathematical research: descriptive, archaeological, mathematizing and analytical. The descriptive methodology concentrates on how mathematics is intuitively used by the members of a community in everyday life, which is exemplified in our study. The methodological approach used in this research is descriptive ethnography. The fieldworker got immersed in the context of the farm, doing, as D'Ambrosio (2004) recommends, participant observations and unstructured interviews. We chose an ethnographic approach because context is best studied through participant observation and in our study the context was expected to play a significant role. Both the ethnomathematical theory and the ethnographic approach require a holistic view of the activities being studied, with special emphasis on the context and culture.

Curiously, most of the studies in the field of ethnomathematics have been carried out in places far away from the European as well as the Nordic countries, for example Harris' (1991) study of aboriginal perspectives on space in Australia, Gerdes' study of Angolan sand drawings (1991, 1994), Zaslavsky's (1973) study of mathematical practises of African people, or Ascher's (1991) study of the Inuit, Navajos and Iroquois in North America. In addition, the studies carried out in Europe have been focused on specific classes or groups far from the rural context (e.g. Frank, 1999 or Oliveras, 1997). In this sense, mathematics education research has ignored the rural context. For that reason, the research was conducted in Europe, on a farm in Lleida, in the western part of Catalonia, Spain. Furthermore, one of the authors is a native from the area, which facilitated parts of the data collection process. Lleida is a mostly rural area whose economy is based on fruit production and animal rearing and the chosen farm is a small cattle farm in which the animals are raised as livestock for quality meat. It has a capacity for 580 calves and about 500 lambs. The farm is not industrialized and is run by three farmers: Jaume, Elena and their son Pere. All of them have some basic education but Jaume and Elena did not finish their primary studies. Jaume and Elena were our informants. The fieldworker together with the two farmers explored the mathematical content of their daily activities, as well as how they made sense of their surroundings.

The participant observations generated notes and audio recordings of the daily activities. Through the unstructured interviews with the two adult farmers, biographical data was compiled and life stories were collected. In addition, the fieldworker took photographs of their activities and daily routines as supportive visual data, that became particularly useful when recalling the different utensils and objects the farmers used in their daily activities.

The fieldworker visited the site several times and the data were collected during the spring and summer of 2004. The fieldwork on the farm consisted of 20 days of observations, which were divided into three periods (table 1). The length and goals of each visit varied.

Table 1. *Fieldwork*

Visit	Date	Days	Data collected and methods	Remarks
1st	14.2 – 17.2.2004	4 days	Photos, participant observation, written field notes.	First contact. Establishment of the situation
2nd	5.6 – 24.7.2004 Intermittent	8 days	Photos, simulated interviews, recorded field notes, participant observation.	Getting acquainted with the general context.
3rd	13.12 – 20.12.2004	8 days	Photos, acting out, shadowing and apprenticeship interviews, recorded field notes,participant observation.	Intensive period. Participant observation and detailed data, impressions.

After each period, we did a preliminary analysis of the data in order to see what else was needed, as well as to get an overview of the process. The first visit was as a period of acclimatisation. The fieldworker went to the farm premises and asked for permission for observation and she used this time as a strategy to minimise the observer effect. For the farmers it was not a problem to have her observing their daily activities and asking questions about it, since the farmers knew her because she grew up in the area. This particular situation gave the fieldworker the possibility to move freely at any time within the farm premises. During the second period of fieldwork, she collected the data by interviewing the farmers and taking notes as an observer. It was an intermittent period, because the fieldworker did not stay at the farm more than two days in a row. In the light of ethnographic theory, the first two visits helped her to collect fruitful data during the last visit. The fact that both visits were short as well as intermittent helped her entrée-adjustment to the farm routines.

The visits softened the odds of having a stranger around and provided the farmers with a more relaxed feeling towards the fieldworker. Longer acclimatisation periods could have been too invasive.

Thus, the last visit to the farm was the most productive and interesting from the data collection point of view. During this last period, the field-worker became a participant observer and did apprenticeship interviews. She was part of the situation in specific moments, in order to come into closer interaction with the farmers, as well as to be able to understand the insights of the activity that they were executing (Spradley, 1980). She also tried to understand the daily activities of the farmers in their environment by examining the objects and physical traces left by them (e.g. tools and marks in the barns), but that was not possible without the help of Jaume.

During the first period of observation the fieldworker did a situated interview (Barth, 1994), where she asked the farmers to tell her what they did when performing their tasks on the farm. She asked in detail about their practices. Throughout the period, when the farmers showed her the farm premises in detail, she did simulated use interviews: the farmers showed her how they would do those things, which they had previously mentioned during the first situated interview. Third, she asked the farmers to show her their normal procedures – as acting out interview. However, the most fruitful interviews were the shadowing interviews and the apprenticeship interviews. During the shadowing interviews, she followed the farmers wherever they went. For the apprenticeship interviews, she asked the farmers to teach her how to do the different tasks. These interviews were a part of her participant observations, where she acted as a farmer and learned how to do most of the activities the farmers had to deal with in their everyday life.

Some of the challenges encountered when applying the various techniques related to the methodology were due to the fieldworker's inexperience as an ethnographer. For example, in a couple of occasions, when interviewing and discussing with the farmers during an activity, her physical position was disturbing their movements. She had to learn to choose the right spot for her observation. Also, while speaking to the farmers, she at times got in the way of their discourses. However, this turned out to be a positive factor, since in their culture it is very natural and common to interfere when somebody speaks. This gave them confidence in her as someone who understood their culture.

We began our analysis by making a list of different tasks we identified on the farm. As shown in table 2, the fieldworker recorded the frequency of incidence and whether the activities were routines or not. After that, in order to systematize the data and analyse it, we created a mind map

Table 2. *Tasks on the farm*

Activity	Person in charge		Frequency of the task		A routine task?	
	Jaume	Elena	Daily	Not daily	Yes	No
Preliminary barn inspection	X		X		X	
Feeding young calves with powder milk	X	X	X			X
Preparation of lamb feed	X	X	X			X
Distribution of grain and feed for older calves	X			X	X	
Distribution of feed for lambs	X	X	X		X	
Cut the hoofs of lambs	X			X	X	
Cleaning the barns	X			X	X	
Stopping ”breast milk” of lambs	X			X	X	
Giving medication (with a veterinary)	X			X		X
House care (cooking, cleaning, ironing, shopping...)		X	X			X
Arrival and departure of animals	X					X
Vaccination	X	X		X		X

with the different data gathered during the fieldwork period. In this mapping, we located the different identified tasks of the farmers and the factors, which intervened, with the development of the tasks. Photos of the tasks were included to complement the data. We compared the field notes with the interview data and memos about the tasks. In the interpretation phase of the analysis we reflected on and considered all the gathered data. The analysis involved connecting the field notes with the information extracted from the interviews.

Solving problems at the farm

During the interviews, the farmers gave detailed descriptions of their daily physical activities as well as detailed descriptions of the different barns and spaces on the farm. Most of the information concerned activities that had already become routine after repetition and experience. However, there were also descriptions of several problems encountered during the course of the activities.

A problem is defined by its objectives or purposes. Hayes (1980) expresses what a problem is by stating "whenever there is a gap between where you are and where you want to be, and you don't know how to find a way to cross that gap, you have a problem". Furthermore the difficulty appears when considering whether an activity becomes a problem-solving situation or not. Bodner (1987) introduced a rather interesting, but simple, way to differentiate a problem from a routine activity. He argued that if one knows what to do when facing the potential problem, it is not a problem but an activity. Thus, most of the activities established in the farm were routinized and therefore not problem-solving situations. Yet some were and we understood them as those activities where the farmers did not know what to do immediately in order to find a solution.

Cattle and their physical condition – such as illnesses, injuries and specific care details – are examples of problems mentioned above. However, there were other problems regarding difficulties related to the daily routines such as feeding and cleaning the barns and also what we call "primary obstacles", which the farmers had to face when doing the activities for the first time, before they had become routines. These "primary obstacles" were particularly interesting because they were mostly related to optimisation and distribution of spaces and time. We chose to focus on these particular problem-solving situations.

Generally, when a problem is encountered and defined, it is developed in such a way that it posits both a clear question and some criteria for recognizing a successful solution. In addition, a strategy for solving the problem is manifested. The strategies can be either executable or unworkable. The final step becomes making the interpretation for the use of the strategy in future tasks.

However, when solving a problem in a formal education situation (i.e. at school), the students tend to rush uncritically (since frequently the problems are out of their school context) to find answers in formulas and pre-established procedures (Schoenfeld, 1985). Basically, those are the tools that their school context provides them with. In the case of the farm, the problems were contextualised in situ and the farmers did not rush into paper-pencil calculations.

We will present two different problem-solving situations on Jaume and Elena's farm and the solutions they found. Both situations are related to animal feeding and their respective solutions involve the use of mathematics. The first problem concerns how Jaume dealt with the use of a barn as a space for the youngest calves and how during the process of distributing the space different mathematical elements emerged. We chose this situation to analyze in more detail because it had been a "primary obstacle" for Jaume and he recognised that it was a problem-solving situation at the beginning.

The second situation is about the use of different objects as measuring tools. We chose it as an example because it is the type of situation where the context does not provide standard tools for solving the problem. Here the farmers had to use what was available in the context of the farm.

Optimisation and distribution of spaces

The farm had an old barn where the smallest calves were meant to live and be fed. Jaume explained that at the beginning this appeared to be a problematic situation because the barn was already built, and thus it was not possible to reconstruct it for serving the new purposes and needs of the cattle. The problem was encountered and defined: He had to optimize the given space and find the variables for maximum capacity. At school a problem posed like this could possibly lead to creating an equation that includes all the variables needed and taking the first derivative of it. Even if Jaume did not have these tools, he still used mathematics to solve the problem at hand.

The only possibility for Jaume was to rearrange and distribute the inside while maintaining the outside structure of the barn. The question was clear: how to distribute the old barn space. The criteria for recognizing a successful solution were that the use of space had to be optimal and the distribution had to permit feeding and living for fifty calves. Jaume articulated the strategies for solving the problem.

During the interviews, he described in detail the process for solving this problem. He drew a plan (horizontal cut of the building) of the barn on a paper and explained how he had examined different possibilities to solve the problem. He tried to draw different pictures for us to understand better the problem and the explanations. He used his intuition to consider the validity of the different solutions he came up with.

He had the possibility to build fences and use a rail of buckets. Jaume had to consider and deduce the possible alternatives from their final purpose. When considering them, he eliminated the ones that were not suitable such as installing permanent fences (since the space was too small and that could reduce the mobility of the animals even more), lining up the feeding buckets from wall to wall (it would have taken too much space and made it impossible to control which animals were fed). All the same, the possibilities of distribution in a square space seemed to be infinite but Jaume insisted that "keeping it simple" appeared to provide the optimal use of the space.

He had to create different models of indoor fencing to find the most suitable one and he placed the fences in different positions to test them (trial-error method), instead of working out the possible models abstractly or on paper (as would have been done in a school). He divided the problem into sub-goals (Mayer, 1983) or in other words, in Polya's (1945) heuristics he used auxiliary problems. The first goal was that he had to differentiate the fed calves from the not yet fed ones during all the feeding process; second goal was to create an area where the animals could eat in peace and still be under control; and third goal was to obtain as big a space as possible for the calves to be kept between feedings. Then Jaume eliminated the obstacles one by one: for separating the fed calves from the non-fed ones, he came up with the idea of putting up fences. The way to get an area where the animals could eat in peace was to create a smaller feeding area, with just sucking buckets. This area's width was a bit longer than the length of a calf, with just enough space for Jaume to fit between the fence and the calves. This was a control strategy as well as it implicated geometrical ideas. The calves did not have room enough to run around. The area was set by the length of 12 calves in a row and the length of a bit more than a single calf, creating a rectangle.

Mobile iron fences were the best choice for Jaume and Elena due to the reduced available space (144 m^2 is not much space for 50 animals). Jaume built them by using two poles with two parallel bars attached in perpendicular to each extreme of the poles and creating a rectangular frame. Jaume claimed that this way the fences were strong and easy

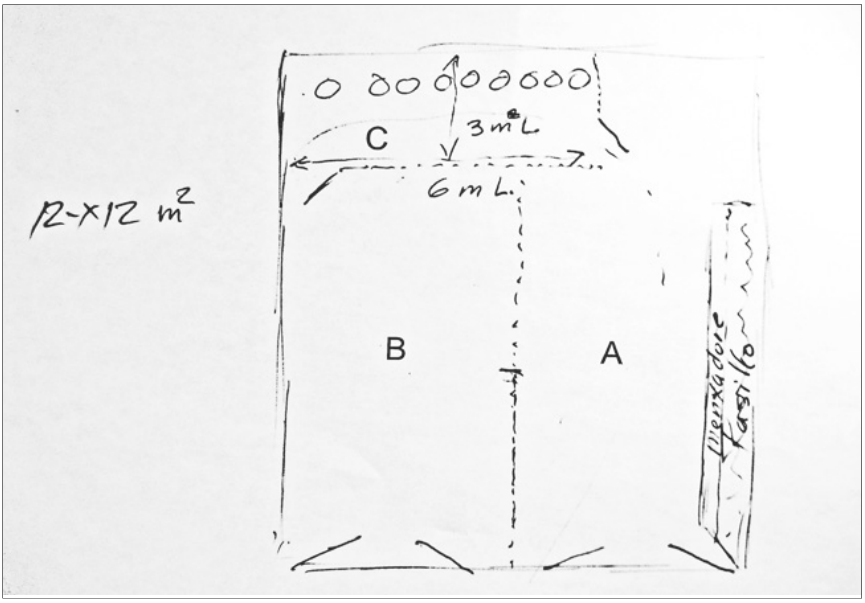

Figure 1. *Jaume's sketch of the barn*

to handle. Once again, he used simple geometrical shapes for obtaining optimal results.

The fences were placed in such a way that the barn was divided in a space for the fed calves (A), another for the calves to be fed (B) and a third one for the calves being fed (C). The spaces were proportional to the amount of calves they were meant to contain. Apart from the sketches that Jaume drew to support his explanation, he also drew alternative possibilities and explained the disadvantages he found regarding the use of space or regarding the control of the animals. Those sketches were very superficial and he did them to clarify his oral explanations. All of them were traced with straight lines and they showed his spatial abstraction capacity. However, when the fieldworker asked Jaume to draft the plan of his own house, he seemed reluctant and argued that that was a more complicated task and he was not able to do it. We see this as a clear example of how much context can modify and provide significance when solving a problem. Jaume used the barn sketch as a tool for his explanations of the problem we were dealing with. All the same, the drawing of a house was not connected in any way to the situation and therefore he claimed that he was not able to do it.

With regard to the space distribution in the barn, the created spaces had different sizes according to the number of calves to be kept. They were proportional. Once more mathematics was used in the solution: proportionality. Space A and B were bigger than C, since C was meant to be for about 10 animals at a time. All three spaces were rectangular (see figure 1).

The transfer of calves from one space to another was done manually by Jaume and in a rotational way.

Jaume: [...] these ones have sucked, they get out from here, and we bring them to this empty space, let's say... well in this place there is nothing. Well we pass 10 more here and we repeat the same thing, the wheel, we make the wheel, all right?

To start the feeding all the animals were gathered in B. From B, groups of 10 calves at the time were transferred into C, fed and then when finished feeding transferred into A. All the fed animals ended up eventually in A. Jaume and Elena explained that they had control over the animals all the time. The animals in A were fed, the ones in B were waiting to be fed, and the ones in C were being fed. After the feeding, the fence between A and B was removed, allowing the animals to have more room to move around. This system was used twice a day, in the morning and in the evening, and it was carried out by Jaume and Elena. Jaume moved the animals and Elena prepared and gave the feed.

As described in the previous fragment, Jaume spoke of the rotational method as "the wheel". The analogy of a wheel to the rotational system is a very clear example of how one's own experience can provide elements for understanding mathematical elements. In particular, Carraher and his colleagues claim that the way humans learn to deal with new specific situations involves remarkable use of previous knowledge such as analogies, categorizations or comparisons (Nunes, Carraher & Schliemann, 1993; Carraher, Carraher & Schliemann, 1988). A wheel represents a round object that serves Jaume as a tool for his reasoning strategy. For him a circle is a never ending object, as the process of feeding is infinitely repeated. He claimed that it was the logical way to proceed as well as the most practical one.

Researcher: [...] but what motives did you have to make this type of rotation of calves (referring to the feeding system)?

Jaume: let's see ... the practicality of it! The system is like this and it is not in any other way ... not any other way. The distribution that we did has to be like that. As I explained to you. Let's say. And it cannot go any other way, inside this barn ... it cannot be any other way. OK?

Researcher: OK

Jaume: But I am really sure that it cannot go any other way.

Above all, Jaume's solution to the problem of distributing the space in the barn and his process for developing the solution indicate Jaume's ability to reason mathematically (Mason et al., 1982). All the way through the process mathematics were in use since he was able to reformulate questions to examine different possibilities, reject or verify them, give examples, describe and deduce, as well as to find conclusions and review the validity of the arguments. Nevertheless, these processes bring us unequivocally back to Polya's heuristics (1945). Jaume understood the problem (how to distribute the barn?); he made a plan and carried it out. He had strategies, he evaluated the advantages and disadvantages (for example the use of fixed fences or lineal mangers) and he came up with a plausible and workable solution.

Transforming everyday utensils into measuring tools

The second type of problem-solving situation has many variations in our data. It shows nicely the fluency of farmers to adapt and to solve problems. During one of the interviews, Jaume mentioned the following.

Jaume: I always give them the measure of milk they need. The measure is ... a pot of milk ...

Researcher:	[?]
Jaume:	[...] a pot of litre and a half of milk.

Jaume used the pot of milk as a standard unit of measurement. The pot of milk was, in fact, an old kitchen pot with an approximate capacity for ½ litre of liquid. Every morning and every evening, Jaume and Elena used this pot for calculating the amount of powdered milk solution to be given to the calves. They changed the use of the pot and instead of being a cooking instrument it was a measuring one. More in detail, they took the pot, which more or less seemed to have the capacity for ½ a litre of water. They made it sure by taking a 1 litre empty bottle of water, filling it up and pouring it into the pot. They emptied the pot and filled it again. There was no more water left in the bottle, therefore, 1 litre divided into two pouring times equals two ½ litres, hence the pot's capacity was ½ litre. After that, the pot was no more a pot, but a measuring recipient with a capacity of ½ a litre. This way, they were able to determine the exact amount of solution they needed for each calf by giving 3 times a full pot to each calf. During the interview Jaume said "a pot of litre and a half a litre of milk", but during the feeding, Elena poured three times the ½ litre pot, and therefore the final amount served was 1½ litres. Jaume had confused the total amount to be served with the real capacity of the serving pot. While discussing with Elena, a similar case took place.

Researcher:	What is the quantity of water that you need? [for preparing milk for 49 calves]
Elena:	4 buckets.
Researcher:	[?] Yes ... but how much is that?
Elena:	... mhm ... well ... I don't know now ... 1 and a half litres per calf.
Researcher:	But in a bucket?
Elena:	Count it ... I don't know ... that is what I take ...

Elena indicated the quantity with a different unit: a bucket. She uses what she has available in the context of the farm to reach her purpose. Conventional instruments are not always available and therefore the farmers had to find another solution. Consequently, in lack of conventional measurement tools they use those everyday utensils that are available and transform them into measurement tools and "standardize" them as measurement units. Apparently, very often, formal mathematics gives attention to subject matter, instead of student skills and strategies to solve problems without tools and means. For example, formal mathematics taught at school prepare students to be able to measure volumes, surfaces and lengths in different measurement units, whereas no attention seems to be paid to figuring out how to measure the same things without standard measuring tools or units. Ironically, often, everyday life lacks those tools and people need to create new and different solutions. Jaume and Elena managed to use tools from their close environment and transformed them into unconventional but reliable instruments. Some of these objects had to be transformed or adapted, like the bucket with sucking teats, which were perforated in their bases to attach the rubber teats and as a result, the young calves were able to suck the milk.

In the farm, we found several similar cases of objects that were used for other purposes. Up to a certain extent, this brings us to acknowledge a double identification of the same object: one as an artifact, the technical device constructed according to specific goals (in the case of the pot, for cooking); and another as an instrument, regarding its modalities of use (Vérillon & Rabardel, 1995).

Reflections: the use of mathematics as a constructive process

One of the first questions that the fieldworker asked Jaume and Elena was whether they used any mathematics in their daily activities. Jaume answered affirmatively and showed the fieldworker their office, where they dealt with the administrative papers of the farm, and pointed at the computer. For him, to use mathematics meant to use numbers; in particular, the numbers of the identification of calves, the numbers that showed the amounts of feed and the weight of the calves upon arrival and before they were brought to the slaughterhouse. Metaphorically speaking, numbers kept appearing all over the farm and in the farmers' daily activities. There were numbers when measuring the feed or when counting animals to be transferred to the C space of the barn, as well as for the number of days that the calves were supposed to stay on the farm. Numbers were everywhere.

All the same, the appearance of numbers did not prove that the farmers used mathematics; however, numbers along with the development of solving problems, and putting together rules and using their own experience, made the whole process mathematically constructive. In other words, Jaume and Elena found their solutions, had their routines and learned from their daily activities. They constantly used geometrical shapes and searched for the maximum space to be used, which is another way to solve optimization problems without using derivatives. The ethnomathematical framework helped us acknowledge the value and role of the context in terms of understanding the use of mathematics by Jaume and Elena. However, it was not the mathematics that Jaume claimed to use when he mentioned the numbers in the farm, but the basic mathematics embedded in their activities, such as feeding animals. In addition, the importance of a meaningful context became especially evident when Jaume did not want to draw a map of his house for the researcher since there was no context for doing so.

According to FitzSimons and Wedege (2007), the importance of this type of research lies not only in its explanatory value but also in its potential social use. As Graeber and Campbell (1993) consider, it is evident that mathematics is more stable when it is learned in a significant context through the reasoning of one's own experiences and it is reflected in the ability to reason. In this paper, the ability to reason mathematically has been acknowledged as the capacity to reformulate things in different ways, as Jaume and Elena did. Their mathematical reasoning allowed them to examine the different possibilities, reject or verify them, and give examples of possible solutions or similar situations. Jaume and Elena were as well able to describe the problems and deduce consequences, as well as to draw conclusions (Mason et al., 1982). And above all, even the possibilities to be examined or the tools to be used were determined by the farm as a context. They had access only to what they could find in the farm and therefore, the context becomes a decisive factor for understanding the use of mathematics in their everyday life. Without understanding the context, it is not possible to understand, for example, their choices of tools or solutions. We see this bond between mathematics and context as ethnomathematics. In other words, when we are searching for the practices and uses of mathematics in Jaume's and Elena's farm context, we find examples of ethnomathematics.

On the whole, a workplace environment, such as a farm, has distinct advantages in contrast to other settings for the use and practice of mathematical skills. Our contribution to the ethnomathematical research is an ethnography done in the context of a farm in Europe, unlike many of the previous studies done in other parts of the world (for example Harris, 1991; Gerdes, 1991, 1994; Zaslavsky, 1973; Ascher, 1991; Clareto, 2003). In addition, the few studies carried out in Europe have been focused on specific classes or groups far from the rural context (e.g. Frank, 1999 or Oliveras, 1997), where the farmers, with their daily activities, informally used rules and elements of formal geometry (e.g. squares, rectangles, right angles, parallel lines or optimisation of spaces), estimation and even measuring. For many adults, geometry is a topic that immediately makes sense to them and gives them confidence in their ability to learn. Thus even though the results of this study are not generalizable to all rural situations, they might be transferrable considering the context. This study reaffirms the importance of the inherent spatial sense in adult basic mathematics knowledge (see Massachusetts Dept. of Education, 1992).

In addition to all these reflections and in relation to the knowledge and behaviour focused on quantity, structure, space and transformation along with problem solving used on the farm, all the findings could be compacted into three different mathematical domains hidden in the activities of the farmers: the measuring domain, the numerical and quantitative reasoning domain, and the geometrical-spatial reasoning domain.

In the measuring domain, Jaume and Elena had to calculate the volume of pots and other instruments they used in their daily activities as well as the doses of medication for vaccinations or the amount of feed served to the animals. They used different containers and pots; however, all of them were homemade and self-created, like the pot to measure the milk replacement solution.

The numerical and quantitative reasoning domain was exemplified when understanding and using numbers for different purposes, or when counting fed calves or calculating the amount of feed ingredients.

Finally, the geometrical-spatial domain included the different geometrical approaches when distributing and dealing with the limited space they had in the barns for their different activities, for instance, cleaning the barns, feeding or vaccinating.

The participants of this study, Jaume and Elena are clear examples of adults who reason intuitively (see also Coben, O'Donoghue & FitzSimons, 2000), with common sense. They base their reasoning upon experiences within the specific context of the farm and they use a variety of methods to solve their problems. They benefit from ethnomathematics. The utensils and spaces the farm provides as well as the timing and the circumstances define and shape the responses of the farmers and the farmers' use of mathematics. Although Elena's and Jaume's skills in formal mathematics might be limited, and the level of mathematics they use in their daily life might be somewhat unsophisticated, their basic numeracy allows them to solve real-life, meaningful problems in rather complex context-specified situations. As mathematics educators we could be more sensitive to the fact that human beings make sense of their environments and we could learn more about the ethnomathematical idea of contextualized mathematics.

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II

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ORIGINAL PAPER

Cabinetmakers' Workplace Mathematics and Problem Solving

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Abstract This study explored what kind of mathematics is needed in cabinetmakers' everyday work and how problem solving is intertwined in it. The informants of the study were four Finnish cabinetmakers and the data consisted of workshop observations, interviews, photos, pictures and sketches made by the participants during the interviews. The data was analysed using different qualitative techniques. Even though the participants identified many areas of mathematics that could be used in their daily work, they used mathematics only if they were able to. The cabinetmakers' different mathematical skills and knowledge were utilized to their skill limit. Cabinetmakers were found to constantly face problem solving situations along with the creative processes. Being able to use more advanced mathematics helped them to solve those problems more efficiently, without wasting time and materials. Based on the findings, the paper discusses the similarities and differences between problem solving and creative processes. It is suggested that the combination of craftsmanship, creativity, and efficient problem solving skills together with more than basic mathematical knowledge will help cabinetmakers in adapting and surviving in future unstable labour markets.

Keywords Workplace mathematics · Problem solving · Creative process · Jigs · Cabinetmakers

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Introduction

Working life and the needed know-how at workplaces is changing all the time, not least because of technological advances. These change processes, along with the constant demands of efficiency, questions what kind of skills and knowledge are needed to manage or succeed in working life, and what should be taught in vocational education. The question is whether the focus should be just on the context-bound knowledge needed at a specific work. From the point of efficiency, it could sound reasonable. However, there are scholars (e.g. FitzSimons 2014) who remind us that the only significant issue at work is the constant need to learn things and solve problems that do not yet exist and for which we do not have any prior experiences. To be able to solve that kind of problem professionals must produce and use new kinds of knowledge and reproduce the old ones. According to FitzSimons, many such problems need creative and innovative solutions where mathematical knowledge has a significant role. That is why research should focus more on how various emerging and even unexpected problems are solved in workplaces and on exploring what kind of mathematical knowledge is activated in those processes.

In this study, the aim is to find out what kind of mathematics is needed in cabinetmakers' everyday work and how problem solving and finding solutions to emergent problems are intertwined in it. The cabinetmakers' profession is located interestingly between the old, traditional handcraft methods and new technology. According to the Ministry of Economic Affairs and Employment (2015; Tuomaala 2016; Occupational Barometer 2018) in Finland the cabinetmakers' profession is one of those that are at great risk of unemployment in the near future and this fact does not seem to be unique elsewhere (Frey and Osborne 2017). Being an important part of the Finnish wood industry, mass-produced furniture is the outcome of business expertise and engineering skills combined. At present, modern wood factories employ all sorts of specialized workers in the different product elaboration phases (e.g. assemblers, machinists, hand-sanders, finishers). However, the basics of furniture production will always be a craft-based industry due to the use of a natural material. Prototype-work is in any case based in craftsmanship. Hence, some cabinetmakers will still be needed in the future (Ministry of Education and Culture's working group on increasing the competence of and educational opportunities for the unemployed 2017). The question of who will survive in the future unstable labour markets is raised.

This paper is structured in the following way. First the main research done in the field of workplace mathematics, problem solving and creativity is outlined. Second, the research questions are brought to light and the methodology of the work is described. Third, the different circumstances of each of the participants are revealed and the paper continues with a cross analysis of the core subject matters -i.e., mathematics in use, problem solving and creative process. Last and based on the findings, the similarities and differences between problem solving and creative processes are discussed.

Workplace Mathematics under Consideration

Both mathematics and workplace are terms embracing profound crucial interpretations of their meaning and effect. The workplace is the site at which a person produces work and it might be located in any place where work is performed including homes, offices, manufacturing facilities, farms, stores, workshops or outdoors. Workplaces have existed for a long time and they will perpetuate in the future but modified and adapted to the moment, as it has happened until now. Therefore, the different settings, practices and dynamics embedded in the workplace have been of great interest in different research fields (Malloch et al. 2011; Arminen 2001; Virolainen 2007; Pajarinen et al. 2015). On the other hand, mathematics is known as well to be everywhere around us. Accordingly, more and more studies have been driven to inspect the influence and impact of mathematics outside the formal setting of the school and consequently, in the workplace (Moreira and Pardal 2012; Saló i Nevado et al. 2011; Zevenbergen and Zevenbergen 2009).

The combination of both concepts generates an attractive fundamental outcome and it is the reason why, at least over the past two decades, researchers have been keen on studying different workplace settings and the mathematical concepts and processes used by different professionals. For example, Pozzi et al. (1998) studied paediatric ward nurses dealing with ratio and proportion problems and discussed the implications of workplace practices and emphasized how valuable are the informal strategies used in the ward. Also, Saló i Nevado et al. (2011) explored how farmers dealt with distributing the space in a barn to feed calves and how they used different items as measuring devices. Their study reassured the significance of spatial sense and how basic numeracy allowed the farmers to succeed in rather complex context-specific situations.

Earlier, Milroy (1992) focused on the carpenters' geometric ideas and strategies and ratified the tacit mathematical knowledge in the carpenter's actions. In her study, the mathematics at work were considered from the point of view of the participants and she documented mathematical concepts and processes such as spatial visualization, proportionality or symmetry. In another ethnographic study Moreira and Pardal (2012) examined masons' professional practices in Portugal aiming to illustrate the mathematics embedded in the daily practices of the masons. Their work described in detail how geometry and arithmetic emerge from the masons' work tasks.

Some researchers have attempted to view the mathematical practices at work through the eyes of school mathematics. An example of this is the project of Hogan and Morony (2000) where teachers were sent to find mathematics in different workplaces. The study gathered their reflections on different aspects of the research such as the impact on their thinking, doing research and mathematics in the workplace. The teachers were sent to the workplaces, shadowed workers for one day, conducted an interview and wrote about their findings (2000, 101). The project revealed that the teachers were taken aback by the amount of mathematics found in the workplaces and the mathematical skills displayed by the workers. Bessot (2000) questioned whether it is admissible in teaching to transfer mathematical knowledge that has been shaped and altered at the workplace. She looked into how construction builders constructed temporary moulds to build a wall on an inclined slab and contrasted the mathematical knowledge used to that transmitted by teachers in high school. She alleged that in a construction site there are further considerations to be made before using something mathematically. She mentioned two aspects: one is 'the anticipation' of the actions to be used and the second one is the 'verification of the result of the actions' used. These two aspects are not always self-evident in the mathematics taught in French high schools, since they are not visibly needed for the students. Teachers are aiming at the practical use of the mathematical knowledge. Often the conditions of the reality, where the mathematical knowledge might be used, do not allow such applications.

Magajna and Monaghan (2003) used Saxe's four-parameter model (Saxe 1991) to examine the work practices of glass factory technicians. Their study resolved that while good understanding of mathematical concepts is often required, most significant is to be able to relate the mathematics to the context (2003, 121). Saxe's model was developed to elucidate mathematical practices in a cultural transition and it was focused on emerging goals under four parameters as in activity structures, social interactions, prior understandings and conventions and artefacts. Saxe applied the model in studies of street-sellers' practices.

In terms of mathematical content, there are studies that claim a constant appearance of mathematical elements such as proportionality, approximation, basic geometry, etc. (Greiffenhagen and Sharrock 2008) and not only basic arithmetic (Williams and Wake 2007; Straesser 2000) or simple algorithms (Riall and Burghes 2000; Hoyles et al. 2001). Thus, it is clear that mathematics is embedded in countless diverse workplaces. However, up to certain extent what early studies before the 1990s seem to disregard is that mathematics is much more than the use of arithmetic or geometry (Cockcroft-report 1982). Mostly the studies mentioned up to this point have looked at the specific mathematical knowledge and some of the mathematical practices. In other words, as the literature review shows, researchers in this field have shed light on various practices, mathematical concepts, contents and tools that are embedded in different professions. To some extent, previous studies show how school mathematics and workplace mathematics differ from each other; even though, one of the primary goals of mathematics teaching and learning is to develop the ability to solve a wide variety of complex mathematics problems that may occur at the workplace (Stanic and Kilpatrick 1988).

FitzSimons (2014) asks what actually is vocational or workplace mathematics. According to her in todays' context of globalization and rapid technological, social, economic and environmental changes, the most or even only significant issue is the constant need to learn things that do not currently exist, and to solve unexpected problems for which there are no any prior experiences. In order to solve future problems, one must be able to produce and use new forms of knowledge and re-contextualize the old, existing ones. These kinds of problems are likely requiring creative, innovative solutions, where mathematical knowledge has critical role to play. That is why, research should focus more on how people find solutions to various, even unexpected problems that emerge in workplaces and to explore what kind of mathematical knowledge is activated in those processes.

Problem Solving and Creativity

A problem is by definition something that one does not have the experience to solve (Resnick and Glaser 1976) or when a person has a given aim, but he/she does not know how to reach it (Duncker 1945). Accordingly, Mayer (1990) defines problem solving as the collection of the cognitive processes that take place when transitioning from the current state where one does not know what to do to the final state where a solution is found (as cited in Csapó and Funke 2017, p.62).

Correspondingly, when past experiences are enough for dealing with a *problem*, it cannot be considered a *problem* and it becomes an *exercise* or a *task* (Liljedahl 2004). When solving a *problem*, one makes use of past experiences in addition to direct efforts and a sudden inspiration what Hadamard (1945) would call illumination in the creative process. It is at this point where the *problem solving process* and the *creative process* get intertwined and, up to some extent, fused (Van Harpen and Sriraman 2013). In many studies, the distinction has not always been obvious and, for example, in studies entirely aimed at creativity, the participant's account of the creative work process is labelled as "an open-ended process without a clear direction to an end" with an unlimited time commitment (Taylor 2012, 49), which is basically a definition of problem solving. Also, other studies use the terms as if they were synonyms (see for example Lubart 2001). Nonetheless, for this study it is an imperative to separate and differentiate these two concepts as Wimmer (2016), who in her short essay argues that "successful problem solving can be regarded as a sufficient condition of the creative process". In this study, problem solving is understood similarly to Mayer (1990) conception, as the transition process in which a person with a specific aim, shifts from the present stage of not knowing what to do to finding a valid solution and executing.

According to Liljedahl and Allen (2013), the different understandings of what problem solving is may be summarized in six divergent lines. The first one is problem solving by design, where prior knowledge and experience shape the process of the problem solver and infer the chosen strategies (Bruner 1964). The second line is Pólya's Heuristics (1957) and the four stages of problem solving: understanding the problem, conceiving a plan, executing it and reflecting over it. Up to certain extent, this line is a polished version of the problem solving by design, since in order to succeed in the four stages, once again one must rely on prior knowledge and experience. In the third place, Alan Schoenfeld distinguished different strategies that individuals use spontaneously (Schoenfeld 1983). He defined problem solving as a process where an individual's prior knowledge, actions and views collide, emerging within a certain context. Fourth, is Perkins' "breakthrough thinking" (2000) where problem solving is a process that depends on extra-logical kick that he calls "breakthrough thinking". In this process, the individual must first admit being stuck without a strategy and proceed to what he calls introspection. Fifth, Mason et al. (1982) present another line in "thinking mathematically". For Mason et al., problem solving involves the processes of specializing and generalizing. Specializing is presented as a phase in which the individual gets to know the problem per se. Generalizing is understood as the part of the process when solutions are tested. According to Liljedahl and Allen (2013), the sixth and last line in problem solving is the gestalt psychology of problem solving, which defends that problem solving cannot be taught since it is a product of insight (Koestler 1964) and that a problem may be solved by turning it upside down over and over (Liljedahl and Sriraman 2006). The main criticism of the gestalt's vision is that the inside moment is unattainable and cannot be researched.

On the other hand, the conceptual framework of the creative process emerged from Wallas (1926). His model was linear and had four stages: preparation, incubation, illumination and verification. Hadamard (1945) redefined Wallas's model while working on conceptualizing the process of mathematical invention (see Sriraman (2004) for other creativity models) and transformed it into a stage theory (Liljedahl 2009). For Hadamard, Wallas' stages embraced the whole process of creation including unconscious phases. Initiation is the stage where the first consciously intended work takes place. It can be regarded as the first encounter with the problem and where the setting is compared with past experiences while searching for a solution (Bruner 1964). In the second stage, regarded as the incubation stage, the person stops working on it at a conscious level (Poincaré 1952). The third stage is the illumination stage where the unconscious bonds with the conscious in a brisk of lucidity of a possible solution. Liljedahl (2004) regards it as the "AHA! Experience". Verification is the fourth and final stage where the suitability of this emergent idea is evaluated. In this article, Hadamart's redefined creative process model of Wallas linear one is used as a broad frame to start the analysis.

Research Questions

This study starts with the assumption that it is not possible to work as a cabinetmaker without some mathematical knowledge. In order to reach the aim of finding out what kind of mathematics is needed in cabinetmakers' everyday work, the following question is posed: What are the mathematics in use needed by cabinetmakers? To answer this question, the study first explores the mathematics needed in everyday work that is identified and labelled as mathematics. Since this question is mathematics-based, the study looks at work through the "lenses of mathematics" from both perspectives, as a participant and as an outsider.

Accordingly, to reach the aim of finding out how problem solving is intertwined in cabinetmakers everyday work, the following questions are posed: What are the typical problem solving situations faced by cabinetmakers and how does the problem solving process proceeds? To answer these questions, problem solving situations faced in cabinetmakers' work are considered. By them, the study refers to the challenging, problematic situations in the work process, which must be solved and need solutions and acts to proceed to the next stage. The starting point is the work itself, the problems emerged and how the cabinetmakers find solutions to various, new, even unexpected problems that emerge in their workplace.

Methodology

The Participants and Their Workplaces

The informants of this study are four Finnish cabinetmakers and their workshops represent the context of the workplace. The three different workshops were located in the metropolitan area of Helsinki in Finland. One of the workshops was situated in a vocational school and it was used for the teaching purposes as well. The workshop was well equipped and had modern machinery. The second workshop was a reformed old farrier workshop with traditional and old-fashioned machinery as well as modern. Several craftsmen used this workshop during their free time and for personal projects. The third workshop was a rented space from a warehouse where several cabinetmakers and companies had workshops. Here different tailor-made furniture was produced.

All four participants were Finnish male cabinetmakers, from 38 to 65 years old, with the same vocational school training. In the Finnish educational context, it means that they have studied mathematics a minimum nine credits out of a total 180 (Finnish National Board of Education 2013; Opetushallitus 2016). Each of the participants had experience in the labour markets. They either had their own company or worked for someone else. All of them were respected and skilled craftsmen in their field. For the research purposes the participants were named Jacob, Thomas, Anthony, and Frank.

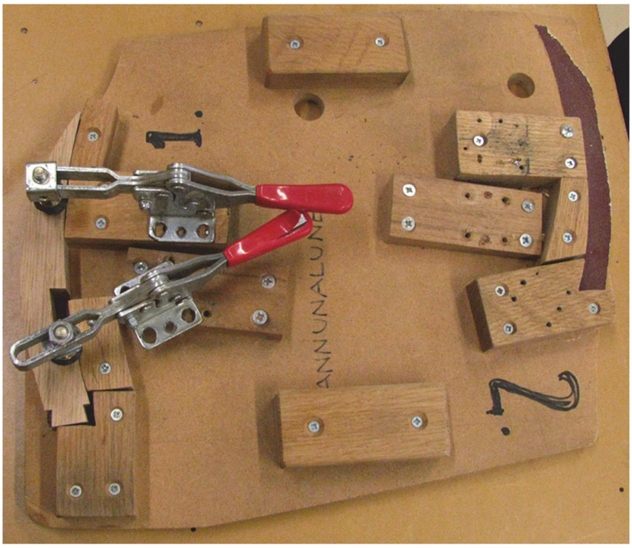
Data Collection

An ethnographic approach was used in the data collection, which has been pointed out to be an appropriate methodology when trying to understand mathematics from participants' point of view (Barton 1997; Hodson 2004; Atkinson and Delamont 2005). The main data consisted of workshop observations with fieldnotes, interviews, videos and photos. In addition, the data was completed with pictures and sketches made by the participants during the interviews. The data was collected in three phases.

During Phase I, the data was gathered via workshop observations and several individual semi-structured interviews of each participant (Rapley 2001; Atkinson and Delamont 2005), where the cabinetmakers were asked to describe in detail their daily routines and tasks at work. They were invited to have the first interview with open questions in anticipation to guide the conversation such as *'Please, describe an average day at work' (to get an overall description of a typical day), 'what do you do when you get here?' (to get a more detailed list of actions and happenings upon arrival to the workshop)*. The sites were visited several times. Each cabinetmaker decided the place to be interviewed and, except for the first interview with Jacob, which took place in his own house, the rest of the interviews were conducted in the respective workshops of the cabinetmakers. All the interviews were audio recorded and later transcribed. In the interviews, some questions were made to elicit detailed descriptions of the cabinetmakers daily routines and tasks: first some general questions about the cabinetmakers background and education in the field and gradually more exhaustive questions about their tasks and details of their job (such as '*what do you do when you get a client contacting you? Please walk me through each and every step?', 'What do you do when you deal with something else than 90 degrees angles in a piece? Can you show me?'*).

Phase II of data collection took place after the initial analysis of the data collected in the phase I. Its aim was to focus particularly on how the cabinetmakers conceived the problem solving situations. During the initial analysis of the Phase I data, it was found that "making jigs" was a typical problem solving situation in the participants' everyday work. Therefore, the phase II had a targeted approach since it was needed to better understand these situations. In this phase, the cabinetmakers were asked to show different types and examples of jigs and explain their uses (Pictures 1 and 2).

Jigs are self-constructed appliances for guiding the machinery or supporting the assembly in a specific stage of the job (Paavola and Ilonen 1981). In other words, jigs are aids in the working process and typically needed for a unique situation. The informants were asked about the process of creating those jigs. Since each jig is related to a project process, several projects were pursued, for example: a tool closet door, a trivet, a wooden sandal, a wardrobe or decorative wooden icosahedra. All the jigs needed to build a four-sided trivet and a pentagonal trivet, were discussed. Field notes,



Picture 1 Two jigs in one board (numbered) for making the arms of a trivet. In jig number two it can be seen how is the piece of wood fastened

researcher reflections and memos were collected during the observations. In addition some photographs were taken and videos were recorded mainly collected to support the interview data.

Additionally, the Phase III of data collection was meant to document a cabinet-makers project from the very beginning until the end, to see the spontaneous appearance of problem solving situations. Jacob was asked to take part in the project documentary where he was to contact the researcher every time he was going to advance in the development of the project. The data was video-recorded and the shadowing interviews (Blake and Stalberg 2009; Quinlan 2008) were unstructured with open-ended questions such as *Could you tell me what you just did?* or *Can you put*

Picture 2 Jig to guide the router when making a hole. In the picture, the router is being guided by the jig (wooden plank with a hole)

in words what you just did to create that ellipsis?. They aimed to obtain descriptive data of the cabinetmakers' performance. The video-recorded data and the photographs helped to recognize the mathematics that the cabinetmakers were not able to see by themselves. For this piece of the study, the Phase III data provided additional and more sharpening data of the jig making.

Data Analysis

In the beginning, the collected interview data were transcribed and an inductive qualitative data-analysis (see Thomas 2006) was applied. It meant detailed reading at the raw data of and - in this case - also looking at it. The derived concepts and themes emerged from the data.

Hence, the analysis started by close reading of the interview data and fieldnotes from observations, writing memos and summary sheets and coding the photos and so that they could be connected to the other data. Identifying the emergent themes was the next preliminary phase in the analysis. The themes were used to reach preliminary order in the data and to help understanding of the cabinetmakers' work and to advice what type of data was needed to obtain in the following phases. The identified nine themes (work strategies, daily routines, technology, experience and skills, feelings, tools, working processes, problems and materials) emerged in all the interviews. The cabinetmakers explained thoroughly their daily tasks, how their day was organized and what type of jobs they had to do. A big part of their descriptions ended up as examples of how to use tools and technology and how to handle materials for optimizing the results. When describing the working processes, they explained accurately all the stages of their job starting from the point when a customer makes an order. Along with these descriptions, feelings and experience of past projects were manifested. The data indicated that problem solving had an inevitable role in cabinetmakers' everyday work. Typically, the problem solving emerged in a situation when a needed jig was to be constructed. Since the jigs typically are unique tools they need to be "invented" in the construction process. So, it was an imperative to consider the linkages between problem solving and creativeness, and to obtain more new data about problem solving.

The analysis continued during and after the data collection by ordering the thematised data under the topics of *mathematics*, *problem solving* and *jig creation*. The *mathematics* was further analysed thematically identifying various use of mathematics in the cabinetmakers daily work. Through the "lenses of mathematics" the use of basic calculations, percentage, measurement, estimation, geometry and trigonometry could be derived from the data. Some of the use of mathematics the cabinetmakers were able to identify and label by themselves as mathematics, some could only be depicted from the observations, fieldnotes and videodata. In addition, the different mathematical skills and knowledge of cabinetmakers became evident. Collaborative data analysis (Cornish et al. 2014) was applied through the entire analysis process. Concerning *problem solving* and *jig creation* the significance was attached to the different stages of the processes and by refining the understanding of the cabinetmakers' procedures. To sketch the problem solving and jig creation procedures a huge poster-type sheet was produced with the data from observations, fieldnotes and photos linked to the interview data. The collected video-data was systematized by extracting different excerpts that deepened the different notions that appeared in the cabinetmakers' interviews. The researchers read carefully ordered data, made first independent interpretations and later on discussed them to reach shared understanding and interpretations. In addition, the cabinetmakers were given the opportunity to consider the interpretations, and some minor modifications were made.

Contextualizing the Research

With-the-Grain Approach

In the world of cabinetmakers, when working with wood, 'With-the-grain' cuts are done on the wood parallel to the long axis to expose plain grain. In this paper, the title "with-the-grain approach" is used as a metaphor, since the aim is to 'expose' and describe the different cabinetmakers' settings at the time of data collection.

Jacob

Jacob was 38 years old at the time of data collection. He had his own workshop, but he was not working as a full-time cabinetmaker. He had over 20 years of experience in the field and made tailor-made pieces and chose his customers according to the time and the amount of work. He worked in the old farrier workshop that he had reformed and adapted for his needs as a cabinetmaker. Jacob considered himself to be a traditional cabinetmaker and he used the term "old fashioned" to explain that he liked to work with the timber the old way, without computer blueprints and making joints without screws or nails. Jacob enjoyed working with his hands and liked to feel the wood, establishing a dialogue with the different timber he used. He loved to touch and manipulate the pieces in his hands while he got lost in thought. When Jacob got excited about something, he took pleasure in thinking about it over and over and maturing the idea for a long time before taking action. This made him a perfectionist and resulted in using a lot of time in his projects, always finding room for improvements. Jacob did not easily give up and he tried and tried repeatedly until obtaining the desired result. Regarding collaboration with other cabinetmakers, Jacob kept a small intimate circle and shared his ideas only at one-to-one level.

Thomas

Thomas was 47 years old and he had almost 30 years of experience as a cabinetmaker. He ran his own company with several workers and his workshop was a rented space in a warehouse where other cabinetmaker firms were located. In his workshop and through the years, he had been collecting diverse tools and machines, which he considered to be life-long investments, particularly jigs of different past projects. Thomas had a vast experience as a cabinetmaker and described himself as traditional in his methods and ways to work. He claimed to love mathematics, but he refused to use advanced mathematics (i.e. trigonometry) and computers in his daily tasks. According to him, basic mathematics in addition to trial and error repetitions did the job. He was social and sought human contact while working; for example, he valued the coffee breaks outside the workshop engaging in conversation with other cabinetmakers. Thomas described those as moments for thinking and, for him, spending time thinking about something was a crucial element in any process. Thomas often had apprentices at the workshop from different vocational schools. He liked to pose problems for them, for example, give the apprentice a model of a perfect wooden icosahedron and ask him to replicate it. The apprentice could spend several days or even weeks looking for a way to do it. Thomas claimed that learning by doing was the most important thing to build up a good foundation of experiences for the future and when experience would fail, a conversation with others may enlighten some sort of solution.

Anthony

At the time of the data collection Anthony was 40 years old and he was working as a cabinetmaker teacher in a vocational school. He had been in the field for almost 20 years and he used the workshop of the vocational school to work for his own projects outside his working hours. Anthony was serene and patient while working. He was keen on experimenting with other materials such as metals and he fully relied on and used different computer-based machines. He knew how to use mathematical knowledge and he applied it every time he had a chance in order to be more effective and exact. He was able to use different computer programs and the blueprints for the jobs. Anthony was happy to explain and share the knowledge and reasoning behind his actions and at all times he seemed to be able to link it to the mathematics behind each procedure and tool. He was curious at times, a good observer and at the same time eager to start a conversation about the pros and cons of a detail.

Frank

Frank was the oldest of all. He was over 60 years old at the time of the interview, but he had finished his studies as a cabinetmaker recently. He had more than 5 years of experience. Frank was using both the vocational school workshop for bigger projects and Jacob's old farrier workshop for smaller ones. Frank was unperturbed, quiet and reflexive. He was not too keen on discussions but enjoyed a friendly talk with a colleague. Often Frank wanted to check his procedures with other more experienced cabinetmakers. He was rather traditional in his taste but reluctant to do things he was not acquainted with. He did not utilize advanced technological machinery and blueprints since he felt insecure operating them. In other words, he preferred to use secure and well-known procedures rather than to risk using unfamiliar methods regardless of the perfection of the outcome.

Across-the-Grain Approach

In the world of cabinetmakers, when working with wood, a cut across the face of a board will reveal end grain. Likewise, this section is named "across-the-grain approach" as a metaphor since the research pulls from across the settings of the four cabinetmakers themes pertaining to problem solving at the workplace and therefore revealing the findings.

Mathematical Knowledge in Use

The idea behind this study was not to claim an innovative mathematical behaviour of the cabinetmakers, nor to discover a new use of mathematics. It begins with assumption that cabinetmakers used mathematics (Milroy 1992; Greiffenhagen and Sharrock 2008). The data supported this initial premise. In the interviews the cabinetmakers described and identified the possible mathematics in their work. It also was depicted from workshop observations. In the following quotation Anthony listed the possible mathematics faced in the everyday working situations:

> "*Of course (I need mathematics), when I estimate the price for the customer, I must use adding, subtracting, multiplying, dividing and also percentages… I would have to use percentages… and I usually work with fractions and then when I plan the work, I would have to use some geometry…. Also work with trigonometric functions and the percentage again… also when I work with the finishing materials, different kind of… you know, paints and stuff… then I'll have to estimate percentages and amounts and when I use pressuring tools, I have to count pressure… which is mainly multiplying…*" *(Anthony)*

In the daily tasks, all the participants identified the constant use of basic operations such as addition, subtraction, multiplication and division. These were needed for example when measuring pieces, cutting boards, assembling, gluing, making joints and even hole drilling. However, because of the different data types it was possible to depict also the use of mathematics that was not identified by the cabinetmakers. The measurement, which is an essential mathematical dimension of cabinetmaking, could be depicted only vaguely from the interview data. On the other hand, it became clearly visible in the observation data. For example, one of the videos showed Anthony describing the process of making a dovetail joint and all the measurements he needed to consider during the process. Another video recorded Jacob showing how he would measure where to drill the hole to install the leg of a table. In both cases, basic operations dominated their descriptions. Most of the time, the measurements needed to be transformed and operated on before being used in the next step. Interestingly, the participants seem not to identify it as "mathematics". All the participants claimed that the simple basic mathematics is sufficient in everyday work, if they did not face situations that require breaking the routines. "*I think they are very basic operations. Like if I get the le…If I know the maximum length, the main measure and I know that the front frame should be… 30 millimetres shorter from both ends. I must make a subtraction. Multiplication and division and that is really enough*" *(Jacob).*

When Anthony was asked about the mathematics of making joints, which is an important operation at their work, he replied: "*It is kind of easy math actually, mainly subtracting and adding*".

When estimating prices and taxes, the participants needed to count percentages. These were needed also for calculating the amount of various substances to be used when finalizing the pieces.

> "*When I work with the finishing materials, different kind of… you know, paints and stuff… then I'll have to count percentages and estimate amounts*" (Antony).

All four cabinetmakers told about and showed the use of proportions when doing, for example, the measurement drawings of a piece to get a glimpse of how the different parts look when assembled or for dimensioning it. Proportions were also used for proportional reductions or piece enlargements. For example, Jacob mentioned using proportions when constructing a miniature prototype of a piece and when doing its isometric projection. This was particularly seen in the video recordings of Jacob in his workshop.

The skill to estimate quantities and times was very essential to the self-employed cabinetmakers. They must be able to estimate time required to complete the project including the preliminary preparations of the materials (for example the drying the wood), estimate the real prices for handmade products, and needed amounts of materials and components and storing them. Good estimation skills save time, money and materials and they protect the cabinetmakers from making fatal mistakes. Estimation can be quite complicated and exceeds the limits of pure mathematical estimation. Self-employed cabinetmakers must hang onto their old customers, find new ones and consider the consequences of their own actions. Jacob discussed: "*I also have to contemplate how much I want to do this project, because if I realize that this furniture will cost so much that that customer will not ever, ever, never buy it. I can…It's somehow it's mathematics. I have to decide if this is an important way to make a new contact. And if I get this new contact… can I estimate the right prices after this project and get this back somehow. That's the one and …actually it's the most important thing, because cabinetmaker companies are small and make unique stuff*".

The cabinetmakers draw a great deal of outlines and working drawings both for their customers and for themselves. In them and in perspective projections of the pieces they needed plane and 3D geometry. Geometry is very important in many other ways, too. The cabinetmakers calculated areas, diameters, perimeters, volumes and various transformations of them. For this study, an interesting detail was that the amount of timber was often calculated in litres, to avoid decimals. Measuring and calculating the angles were needed – at least in principle – in planning and drawing joints and final pieces, as well as for adjusting the blades of the saw to the needed cutting positions. Here, the cabinetmakers' mathematical skills and knowledge are put to the limit. Where others insisted that trigonometry was not essential, one of them could apply trigonometry and found it absolutely essential in his work.

> "*We are using trigonometry all the time. It is our alpha and omega. You will always end up in trigonometry*" *(Anthony)*

When one of the cabinetmakers, who claimed not to be so keen on using trigonometry, was asked how he was able to make any other angles than 90° or 45° without trigonometry, he replied*:* "*you can do it by trial and error, you know, but it can be kind of… it is really you would use a lot of material and a lot of time… because you actually will have to make a 1 to 1 size model to see that actually work*"*(Anthony)* When this cabinetmaker faced problems, he would to turn to his colleagues or to a professional (mathematics) textbook for help. In the course of the interview the researcher and the participant were looking at the textbook in question and searching for the formula of the adequate trigonometric function. Then it came out that the participant had no clear idea what he was searching for. He admitted that he would benefit from better trigonometric skills and knowledge:

> *(P) "If I'm in the workshop and have hundreds of pieces, you really can't make 'test-assemblies'...*
>
> *(I): For every single piece. You need to...*
>
> *(P) You have to count and then comes the really, really big problems if you can't do that.*"

In an interview Thomas discussed the upper limit of the need for mathematics: "*Quite seldom… sometimes we … we just had an affair with ellipses…. we ended up with an equation of second or third degree. But very, very seldom and it is only just if you are interested in taking that kind of jobs. So, the trigonometry is sufficient for cabinetmakers. But, of course also in trigonometry … it depends, how you are involved in it. If you want to calculate angles of mitre joints in various pyramids, you can get really hard equations. Then, involuntarily you will end up to equations of second degree. When you have two variables, you cannot avoid it. But, there are not many cabinetmakers who will bother their head with so difficult mathematics*".

All the cabinetmakers in the study identified and used mathematics in they work. However, the findings suggest that it is possible to manage with quite elementary mathematics, even when the cabinetmakers have succeeded in their careers. As Thomas put it, it is a matter if you are "*interested in taking that kind of job*" which in order to be completed require more advanced mathematics - or alternatively - a lot of risk-bearing experimenting. Hence, besides and instead of applying advanced mathematics they turned to slow and resource consuming trial and error –methods. On one hand, the study can try to find some explanation from the participants' different mathematical skills and knowledge. On the other hand, the explanations may lie in the fact that the properties of wood do not work fully in the ideal world of mathematics. That is why, sometimes all cabinetmakers had to accept experimenting – despite of their knowledge of mathematics: "*Wood is wood... and it not always so precise. And if you just count, there remains a hole between the pieces, and you shouldn't let that happen... it is of a better quality if the pieces are together. If you compare that you are very good with trigonometry...you can use it very well, but for some reason it doesn't match. It's more important that the pieces are together.*" *(Jacob).*

Problem Solving at Work

During the process of data collection and preliminary analysis of the Phase I data, from time to time the cabinetmakers faced problematic situations and operations where they did not immediately know how to act and did not have routine solutions (Resnick and Glaser 1976). They also recognized these situations themselves. During the interviews, Jacob was reflecting on his work and defined unintentionally the term problem: "*There is also very often that type of project that you are doing something for the first time and you don't know (how to proceed)*". This is very near to Hayes' (1980) understanding of the term "problem" according to whom a problem is the whole between the present stage and the final goal, when the steps to follow are unknown. From this viewpoint, Bodner (1987) suggested that if the steps are known, the whole setting becomes a task, whereas if they are not known, the setting turns into a problem solving situation. Schoenfeld suggests "*a problem is only a problem if you don't know how to go about solving it. A problem that has no 'surprises' in store, and can be solved comfortably by routine or familiar procedures (no matter how difficult!) is an exercise*" (1983, 41).

Problem solving seemed to be very natural in their daily tasks, therefore inevitable and intrinsic. As Anthony put it "*problem solving is a very essential part of my work as a cabinetmaker. A great deal of my work tasks can be described as problem solving, starting with the customers' needs and ending with post-delivery issues. The most central problem in all designing and manufacturing is integrating outer appearance, functionality and costs*". However, problem solving was not referred as simply a fragment of the cabinetmakers daily routines. The participants regarded problem solving often as the most difficult stage of the process when an unknown procedure needed to be done in order to proceed to the next step of a known process.

Particularly interesting were the situations when the cabinetmakers had to plan and construct "the needed new jig". Jigs are self-constructed and typically unique custom-made tools needed constantly in cabinetmaking. To plan and make the jigs involve many kinds of mathematics. Jigs have mainly two functions during a specific stage of the working process: to hold the work in a defined position and to guide the tool in use (Paavola and Ilonen 1981). Usually, it is not possible to make jigs in a routine way, because each new jig is planned for a certain purpose and it requires a solution just for this unique purpose. The jig must be created. These situations were identified as typical problem solving situations in the cabinetmakers work and they assured that "*Almost in every project, at least one, more than one [jig]… and this is really one way to make art*" *(Jacob*). The following quotation belongs to the fieldnotes taken at the workshop where Jacob was building a design dining table: "*The table-top is ready and now he needs to position the legs in a way that they should not interfere with sitters. In order to fix the legs to the table he needs to drill four holes (mortises) for the dovetails to match, since each leg has a dovetail shaped tenon*". Hence, Jacob had to make four dovetail shaped mortises to fix the legs of the table and for that he needed the router. The mortise was not any simple orifice. Jacob had to make a hole in a defined angle and with all precisely defined mathematical parameters (width, length and depth), so the hole could support and hold the tenon of the leg. To drill just that hole, Jacob had to design a jig that allowed the router to stay in place and make the required characteristics' perforation. This made the situation mathematical and thus, the creation of the jig was as well mathematical. This example illustrates that there were at least two mathematical problem solving situations emerging at the same time, intertwined with each other. One is when Jacob had to give mathematical attributes to the hole. The other one is when, in order to get this hole done, he needed to create a jig that allowed him to make this exact hole.

The efficiency of the problem solving process is not always determined by the cabinetmaker's advanced mathematics' skills. Then again, Anthony claimed that advance mathematical knowledge such as trigonometry might economize time and effort since the exact measurements can be established without the delay of trial and error or estimation.

In the analysis, more or less separate steps were identified in these problem solving situations. The participants told that they first approached the problem by looking for past experiences, either from themselves or from their workmates and put to the test different trials and modifications in practice.

> *"We try to find and remember old projects where we have been with the same kind of the problems and then we put a little bit of extra on top of that*"*...* "*Then probably you will negotiate with your friends who have been in that kind of situations before*" *(Thomas).*

Thomas described how a problem solving situation was often shared and discussed with colleagues and generally it required critical thinking, experience and patience. They also tried to visualize the situation to find the solution. Thomas explained that sometimes he stayed awake at night, merely because finding mathematical solutions to problems fascinated him. When the problem was not solved in this way, the cabinetmakers ceased the conscious trying and thinking of the problem for a while: "*then you sit down and have a cup of coffee*". The solution to the problem might appear "*like a bolt out of the blue*" – even in a very different context where the problem was not consciously kept in mind. When the solution was found, the last and ultimate step was to put the solution into practice. Then the cabinetmakers confirmed the details and assessed the feasibility, practicality and quality of results "*and then at the end, when you have solved the problem, the only thing left to just cut, sand and finalize the surface*" (Thomas).

Problem Solving as a Creative Process

The steps identified in the cabinetmakers' work-based and practical problem solving process in the jig creation are to a considerable extent analogous to stages in the model of creative process developed by Jaques Hadamard (1945) decades ago. As a scientist, he was interested in mathematical inventions and, based on these, he developed a model for the process of invention. Later, it has been widely applied to model various creative activities. In Fig. 1, the process of cabinetmakers' problem solving is drawn together with the creative process as modelled by Hadamard. All four steps of the problem solving process identified in the data flowed along with the creative process in a synchronized manner.

In Hadamard's model, the first stage "initiation" is featured by drawing on one's personal experiences and conscious, goal-oriented working. This is almost exactly what the cabinetmakers did since they first turn to their own or their workmates experiences to find the solution, as explicated in the previous chapter. These findings are consistent with the findings of Liljedahl (2009), where a group of mathematicians affirm that talking with colleagues is of great value when solving a problem, and in his work with both, pre-service teachers and mathematicians (2013) where the role of talking is as well an emergent theme of his data.

If the solution was not found, the cabinetmakers told they cease the conscious and intentional trying (the unconscious stages are marked with a discontinuous line). This second step relates to the "incubation" stage in Hadamard's model (1945). In particular, the flexibility and creative manner found in the cabinetmakers when facing the two first

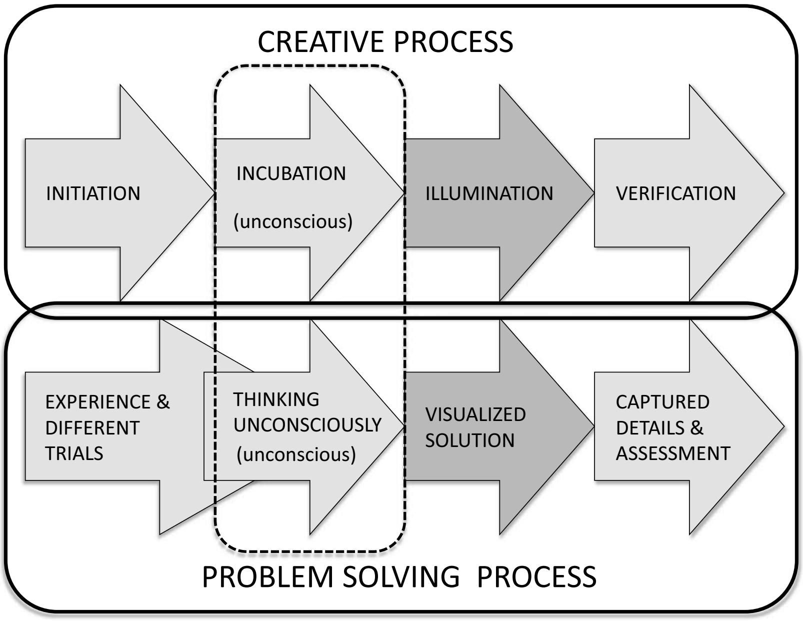


Fig. 1 According to our data, the stages of both the creative process and the problem solving process seem to be analogous

stages of the problem solving situation, namely the creation of a jig, is consistent with research that suggests that the use of mathematics at work is divergent from the mathematics taught at school (Gainsburg 2006; Noss et al. 2002).

Finally, the analysis showed the stage where the eruption of the visualization of a possible solution brings the cabinetmakers the idea and tools to proceed with the task. The third stage of finding the solution (illumination stage) is labelled with a sudden conscious insight "AHA-experience" and it is often loaded with affective aspects of the experience (Liljedahl 2013). This suggests why Thomas expressed excitement and enthusiasm and claiming to *love* spending time creating a solution for a problem. Both Thomas and Jacob described how, at a certain moment during a problem solving process, they were able to visualize the solution. According to them, the visualization was an image that often they would sketch and save as soon as it appeared. Jacob showed several sketches of projects and pieces he had visualized. The last step is labelled as verification stage where the solution will be tested and put into the use. In this stage, the visualization sketches were of great value to proceed in terms of accuracy and perfection.

As shown in Fig. 1, this study suggests that the first two stages of the problem solving process overlap, due to the fact that they are not clearly separated stages and may occur at the same time. The participating cabinetmakers considered that in jig creation the most crucial phase was to conceptualize and to think, visualize and consider what the purpose of that particular jig was. The following extracts from the data refer to the initiation and incubation stages of Hadamard's creative process model and illustrate the need of thinking and planning:

> "*Making jigs is a simple thing. Thinking up jigs is the problem. How do you think them up, not how do you make them!*" (Thomas)

"*Maybe there are manuals, but I think every time you have to plan it and think about how to do it. First of all, what you want to do, what you are going to do and then you'll plan it. There cannot be examples for every situation, never*" (Frank)

Furthermore, the cabinetmakers exhibit a great deal of flexibility during the process and in all the stages, while applying different methods and trials to try to visualize a possible solution to the problems. Often, they discuss with other cabinetmakers and share experiences to try to find a path. Along with the findings of Taylor regarding the creative process (2012), time becomes a key element, since it stretches and it is completely different for each process. For both, our participants as well as for Taylor, time is an unrestricted factor that characterises the processes. Unfortunately, time as a factor is not reflected Fig. 1.

In research literature, the terms "creative process" and "problem solving process" have often been interchanged and used as synonyms most likely because of their similar characteristics, attributes and stages (see Leikin and Pitta-Pantazi 2013; Csapó and Funke 2017; Lubart 2001) as illustrated in Fig. 1. This study considers the concepts to be intertwined, but as they describe different phenomena they should be differentiated. The next section presents the modified version of Fig. 1 to illustrate how the different stages of both processes based on the data are corresponding.

Discussion

The findings concerning the mathematics cabinetmakers identified and used in their work are in line with many other previous studies about workplace mathematics (Williams and Wake 2007; Hoyles et al. 2001; Riall and Burghes 2000). The mathematics they used was in most cases very basic. Interestingly, the cabinetmakers also used mathematics (e.g. measurements and transformations) without self-evidently labelling it as mathematics. Even though the cabinetmakers identified many areas of mathematics that may be used and would be useful in their daily work, they used mathematics only if they were able to. Here, the cabinetmakers' different mathematical skills and knowledge were used to the full. Where one of them was able to use trigonometry, and found it absolutely essential in cabinetmaking, the other thought that it was possible to manage without trigonometry but, admitted needing the help of his colleagues in this respect. The level of cabinetmakers' mathematical skills and knowledge seemed to restrict the choice of projects they could accept to complete. However, it must keep in mind that although the mathematics had a significant role in cabinetmakers' work and helped them to complete more demanding projects, the mathematics as such can never substitute the skilful craftsmanship with the wood.

A remarkable finding in this study was that, for cabinetmakers, the most common problem solving situation is making the needed jigs where mathematics was inevitably intertwined. The findings about problem solving situations led to further ponder about the linkages between problem solving and creative processes. In this case of cabinetmakers, it is noteworthy that one creative process typically included several problem solving processes (building jigs). The stages of problem solving process and creative process share many similarities, yet, as our data reflected and according to Wimmer (2016) they

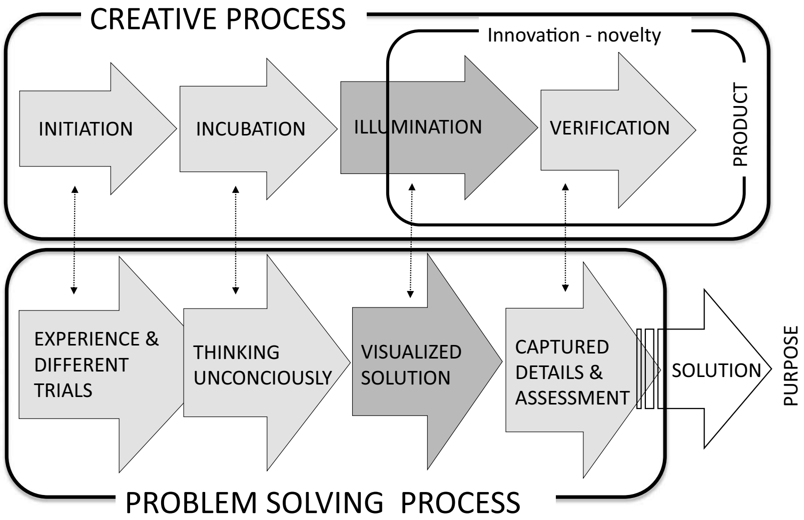


Fig. 2 Problem solving process and creative process based on our data

should not be considered as identical processes. Figure 2 illustrates the similarities and the differences between the processes. During both, the creative process and the problem solving process, the goal is to find, to conceive a final product or solution. However, in the creative process one of the main traits of the final product (e.g. a dining table) or its attributes must be novelty or innovation. Sometimes this novelty has a gradation and may be a mere improvement of a previous product what defines the creativity. On the contrary, in the problem solving process, what matters is the viability of the solution (as it is in the case of jigs). In other words, novelty is a condition of possibility in the creative process as feasibility and practicality are for the problem solving one.

Having said that, depending on its level of novelty and innovation (see Fig. 2), the solution of problem solving may or may not be creative. According to the data, when the cabinetmakers create a jig, the aim is to create something with a purpose and its value depends on its usefulness and not on its novelty (i.e. can the jig hold the piece of wood in the needed position and does it give it room for modifying a specific angle or not). Therefore, the creation of a jig is a problem solving situation and it is not regarded by the cabinetmakers as a creative process, since the jig is meant to (at the same time) serve a definite purpose. For example, when a cabinetmaker wants to design a table, during the process he must invent and build several jigs to be able to make concrete cuts on the timber. These could be considered creative processes but no value is given to them for their uniqueness or originality. Their value is given for their suitability, and therefore, they are problem solving situations within the creative process of designing a table.

In Fig. 2, the solution of the problem solving process is located outside the process box since what is unknown is the procedure and gap between the departure point and the end result. A problem solving process may lead to a creative solution or to a less innovative one, but the validity of the solution does not depend of the level of creativity. On the contrary, in a creative process if there is neither innovation nor novelty, there is no creativity.

The findings and conclusions in this study are based on specific data in a specific context. In the future, more research and various data are needed to elaborate the conceptual and practical differences as well as the relationships between creative processes and problem solving processes. This is the aim in the next stage of this project. The findings reveal that cabinetmakers constantly face problem solving situations along with the creative processes. Although there were no totally unexpected problems in the data, many of those problems were unique and had a number of unknown features. Hence, the cabinetmakers had very little prior experience of them. The cabinetmakers ability to use more advanced mathematics could help them to solve those problems more efficiently, without wasting time and materials. This study suggests that the combination of craftsmanship, creativity, and efficient problem solving skills together with more than basic mathematical knowledge will help cabinetmakers in adapting and surviving in the future unstable labour markets and going beyond the capacity of machinery.

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III

Workplace problem solving within the design process
The story of Pekki table

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This paper presents the story of creating the design table, Pekki – from the initial idea to the manufactured product. The story provides a background for further conceptual analysis. A Finnish cabinetmaker Matti Salminen revealed to us his dream to make a table with distinctive lines. He agreed to be our partner in this study, and we documented the entire project. We closely observed Matti in his workshop, where he explained what he was doing and in the process verbalized his thoughts and actions. We produced field notes, interviewed Matti, informally discussed, made video recordings, took photos and collected Matti's drawings and sketches. Along with the documentation, we attempted to conceptually understand what happened during the process. Our preliminary intention was to examine how problem solving appeared in the process of manufacturing the prototype. It became evident that problem solving situations do not only occur in specific instances of the process, but the process itself was deemed a "problem solving" situation. Moreover, the settings unveiled that creative and design processes were intertwined with problem solving. The aim of this paper is to shed light on and open the difficult ponderations between problem solving in addition to creative and design processes, by answering the following questions: 1. What is the cabinetmaker's process of designing and creating a table? 2. How do the problem solving situations influence the process, and what is the role of the jigs within the process? 3. How are the processes of problem solving, design and creative process intertwined? Based on our findings, we conclude that problem solving has a mediating role between creative and design processes.*

Keywords: workplace problem solving, design process, creative process, cabinetmaker

Introduction
This study is a part of a project regarding workplace problem solving. During the preceding stage of the project we gathered data among a group of cabinetmakers regarding their everyday tasks and problem solving. In the latter stage of our project, we felt the need to observe and document one of the cabinetmakers' tasks from inception to conclusion, in order to explore the problem-solving situations that presented itself. At this point, one of the cabinetmakers, Matti, revealed his intention to craft a table with distinctive lines. At this stage the idea was still floating about in his head, but he had no definitive vision in terms of what it would look like. We documented the process of construction of the table named Pekki, by Matti, the cabinetmaker and how he manufactured the prototype. The preliminary intention of this study was to examine how problem solving appeared in the process of manufacturing the prototype. Cabinetmakers' work can be seen as a combination of technology, engineering and crafts at the mercy of mass production. Sometimes the problems emerging are dealt with and solved by using engineering skills and technology with the exception of prototypes, which mostly remain in the hands of craftsmen. Thus, exploring the problem solving situations and its influence on craftsmanship is a first step to uncover the actual skills and knowledge needed in their work.

Matti agreed to share each step of the process and he contacted us whenever he was spending time doing something related to the prototype. However, the documentation was not linear. The beginning was blurred, and Matti agreed to participate in our inquiry about the problem solving not having a clearly directed idea in terms of what the table would look like or what the process would entail. He had different thoughts and used a considerable amount of time drifting between ideas and inspiration. Our documentation was activated once envisioned and finalized the design. In conjunction with the documentation, we attempted to conceptually understand what had occurred during the process. We realized that problem solving situations did not only present itself in specific moments of the process, but that the process itself was a "problem solving" situation.

Moreover, the setting unveiled different types of processes intertwined, not only problem solving. Firstly, a design process that can be preliminarily defined as the process by which "means" are provided to allow the construction of a new object (Kazakçi, 2013). The design process of a table in itself is a problem solving situation, initiated from the unknown and concluding with the product design. Another process intertwined in the documentation was the creative process associated with the creation of the table, which we understand as the process of generating new and useful ideas with regard to products, procedures or processes (Amabile, 1988). This process seemed rather evident from the beginning, since the dialogue with Matti made it apparent. He was "creating a table".

In this paper, we aim to shed some light on and open complex ponderations about these intertwined processes. More specifically, we aim to determine what role problem solving processes play in creative and design woodworking processes.

Theoretical frame

From Problem Solving to Design and Creative processes

A problem means to find the way to evolve from a present situation to another desired one without knowing the path (Schoenfeld, 1983, p. 41). At work, problem solving is not planned but emerges from the circumstances and settings (Llorente 1996, p. 99). The settings in a cabinetmaker's workshop were distinctive in the sense that often the problem solving was related or linked to mathematics. Problem solving typically occurred when the cabinetmaker had to construct a jig to execute a concrete action with a tool (Saló i Nevado & Pehkonen, 2018). Jigs are self-made tools adapted to and meant to assist another tool or instrument. In most instances they need to be tailor-made for concrete use within a project (Paavola & Ilonen, 1981). Jigs molded the understanding of the cabinetmakers' problem solving by advocating open-endedness. Open-ended (Becker & Shimada, 1997) or ill-structured problems (Jonassen, 2000) are unrestricted to different solutions and therefore, they promote divergent thinking allowing the performance of the subject within the own range of abilities and experience. According to Jonassen (2000, p. 67) ill-structured problems are often encountered in professional practices, and the solutions to these problems are not predictable or convergent. Moreover, the solutions often require the integration of several content domains.

Wimmer (2016) differentiates between problem solving and task solution. A problem has been defined as the gap between the current stage and the final goal, when the steps to follow are unknown (Schoenfeld, 1983). If the steps are known, the entire situation turns into a task, whereas, if they are unknown, the situation converts to problem-solving (Bodner, 1987). Everyday work at a cabinetmaker's workshop facilitates the problem solving aspect in addition to task designs or so-called routine designing components (Saló i Nevado & Pehkonen, 2018). The requirements of the table design in our study guided us towards a design process where the solving procedures are unknown and thus the cabinetmaker is working on a design problem. Jonassen (2000, p. 75) refers to design problems aimed at producing artefacts and he categorizes them among the most complex and ill-structured problems one can encounter in practice (p. 80). Yet, in design, both divergent and convergent thinking are needed (Lawson, 1997) in addition to intuition (Wimmer, 2016). The design of a new product may or may not be creative. We understand the design process as the process of conceiving and attributing an existing item with new traits, features. The product in this study is a table to be commercialized and therefore, an essential attribute of the table is that the design has to be creative and appealing. Nevertheless, our cabinetmaker straddles two categories, that of craftsman and designer (Risatti, 2007). From a technical perspective, he is a craftsman since he has the ability to create and materialize products by hand through the manipulation of the timber in addition to his experience and knowledge of the wood (see Risatti, 2007; Sennett, 2008). Simultaneously, he may be considered a designer in the light of the fact that he possesses an acute sense of form and shape with the aim of producing multiple units. (Risatti, 2007). The object of our study (i.e. the prototype) is seen as the final artefact for a craftsman; yet, for a designer it is the test model for mass production (see Risatti, 2007; Temeltas, 2017). Physical artefacts as prototypes or mock-ups, also called embodiment by Goldschmidt (2017), help the conceptual and material aspects of the design, making the details visible (see Goel, 1995; Dorta, Pérez & Lesage, 2008; Pei, Campbell & Evans, 2010; Lahti, Kangas, Koponen & Seitamaa-Hakkarainen, 2016). In addition to this, sketches and drawings aid the refinement of ideas, shape, or the proportions of the product in the early stages of the process (see Goel, 1995; Lawson, 1997; Aspelund & Kontzias, 2006; Cross, 2011).

Creativity in our case is a precondition for the table as a product. Without creativity in the design, there is no innovation, which would have an impact on the commercial value of the product (Roy, 1993; Howard, Culley & Dekoninck, 2007, 2008; Wimmer, 2016). Emerging then, from the design of the table as a framework and as a problem solving situation, is the need to examine the whole procedure as a creative process. A select few may use the term creative problem solving; however, we agree with Wimmer that it is redundant by definition (2016, p. 3) since a design is creative if the product is innovative, unique, valuable, novel and appropriate (Lubart, 2001). Creativity may be defined as a multifaceted construct including divergent thinking, problem finding and problem solving, originality and efficacy (Runco & Jaeger, 2012) and yet, there is a need for further research to examine the similarities and differences between creativity and problem solving (Wimmer, 2016). In 1926, Wallas presented a four-step representation of the creative process, but it was not until the second half of the twentieth century, when some interest was shown in discerning its origins and how the process works. Different studies have prioritized different aspects of the phenomenon such as the person, the product, the process or the environment (see Basadur, Pringle, Speranzini & Bacot, 2000; Vidal, 2009; Leikin & Pitta-Pantazi, 2013). This is consistent with the elements of the conceptual model of design by Ralph and Wand (2009, p. 108). With regard to the creative person, the main traits are the promotion of divergent thinking, fluency and flexibility, among other traits (Maslow, 1987). Several studies consider how novelty versus expertise of the designer impacts the level of creativity (Gero & McNeill, 1998; Jaarsveld & Leeuwen, 2005;). The ability of a creative person to be intuitive has also been considered as vital component of the creative process (Boden, 1994; Raami, Mielonen & Keinänen, 2010). Furthermore, Raami (2015) argues that intuitive personal experiences are important for the creative process.

With regards to the creative process per se, there have been many attempts to describe the creative process model (see Howard et al., 2008). For a product to be successful the prerequisites are that it is new, useful and original, as well as innovative (Buchanan & Margolin, 1995). Depending on the how much the creative process and product differ from other existing products, radical creativity is distinguished from incremental creativity (Gilson & Madjar, 2011). Wallas (1926) presented a stage model widely recognized but the model was also equally criticized (for more see Lubart, 2001; Leikin & Pitta-Pantazi, 2013). His model distinguishes between four stages: preparation, incubation, illumination and verification. Guilford (1950) claimed that the Wallas model was cursory by failing to observe the mental operations that take place. He pointed out determinate abilities involved in the process such as sensitivity to the problem, flexibility, the ability to deal with complexity and to evaluate (see Lubart, 2001). Another critic of Wallas' model was directed at its linearity of stages (Patrick, 1937;

Eindhoven & Vinacke, 1952). In addition, Sapp (1992) argued for the existence of an additional stage called creative frustration, which might occur between the incubation and the illumination phases.

Over time, Amabile (1983, 1996) presented a model where the phases of the creative process did not occur in an established order. In her model, the phases were renamed as problem/task identification phase, preparation phase, response generation phase and response validation and communication phase. Amabile's work presents a dilemma since both concepts "problem" and "task" seem to be interchangeable, and as was previously observed, task solving processes are different from problem solving processes. However, she was not the only one, since, for example, Lubart (2001) claimed that the creative process models are also framed in terms of problem solving, where a problem is considered a task to be accomplished. Other reviews of Wallas model examined different sub processes of the creative process and later on, some alternative models to the four-stage model were presented. The model presented by Mumford, Mobley, Uhlman, Reiter-Palmon and Doares (1991) was based on a series of essential processes: problem construction, information encoding, and identification of best fitting categories, combination and reorganization of categories, idea evaluation and implementation of ideas. Howard et al. (2008) summarize that the tendencies have veered from characterizing the creative process as subconscious cognitive to activity-based phases.

Research questions

A previous study regarding everyday tasks at the cabinetmaker's workshop indicated the significance and value of problem solving situations, namely jigs, for the cabinetmakers (Saló i Nevado & Pehkonen, 2018). In this paper, we wanted to follow the process involved with a specific job at the cabinetmaker's workshop, and additionally find out how jigs alter it. Therefore, we were looking for an answer to the following questions. Question 1 is a background question for questions 2 and 3:

1. What is the cabinetmaker's process of designing and creating a table?
2. How do the problem solving situations influence this process, and what is the role of the jigs within the design process?
3. How are the processes of problem solving, design and creative process intertwined?

Methodology

Data Collection

In this study, we apply narrative and descriptive methods for data gathering through interviews, informal conversations and shadowing. We focus on a single cabinetmaker, Matti, reporting his experience and telling the story of the Pekki table in detail (Creswell, 2012). The narrated story follows chronological events focused on Matti's story of a table creation.

Shadowing, as a method, is the process where the researcher closely observes the work of a participant over a period of time (Quinlan, 2008; Blake & Stalberg, 2009, p. 243). Shadowing enabled us to obtain a closer understanding of how, when and why Matti the cabinetmaker acted the way he did in the process of designing and manufacturing the table. Shadowing provided a rich data set about his action patterns, motivation, mood, body language and pace of work. Matti agreed to participate and to commit to the shadowing method while working at his workshop throughout the process of designing a table and particularly, building up the first 1:1 prototype. He was ready to tell his story. Data collection was facilitated in his workshop and in his home. The study had a unique circumstance as one of the authors was familiar with Matti and had known him for twenty years. They shared a distinct bond since they engaged in communal leisure activities. From a research perspective, having a solid insider status due to the familiarity component with the research participant, made the shadowing process and the data collection for the narrative research possible (Creswell, 2012) by guaranteeing access and developing trust.

At the time of the data collection, Matti was 38 years old, had his own workshop and worked as a part-time cabinetmaker. He had over 20 years of experience in the field and constructed tailor-made pieces and chose his customers according to his availability and the amount of work. Matti had vocational training, experience in the labor market, and he was an experienced, respected and skilled cabinetmaker in his field.

For the shadowing purposes Matti consented to call the researcher with a 30 minutes' margin to arrive at the workshop to track every single step of the construction as many times as was needed. It was agreed that the shadowing would be captured by video recording all Matti's steps and asking him to verbalize his thoughts and actions. Occasionally additional detailed explanations of concrete actions was requested. The researcher was allowed to follow all the steps and stages without reservations. Due to the risk that Matti could feel judged and evaluated through the shadowing process, the connection between him and the researcher had to be one of absolute trust (Blake & Stalberg, 2009). We gathered additional information derived from Matti's body language, his state of mind and disposition, his time management as well as his working pace. During the process, Matti had opportunities to share his experience, to explain what he was doing and why, since the researcher was present and constantly incentivizing Matti's reflections. The researcher had to adapt to Matti's working spaces, conditions, needs and his ways of working. At times, the workshop was cold and was filled with wood dust, which made it difficult to breathe and record. The lenses of the camera needed to be constantly wiped clean and the hands of the researcher got cold.

The workshop was located in the basement of an old wooden building with limited space, dim lighting and not so efficient ventilation. The space was approximately 32 square meters and had an adjacent room of 70 square meters, where the band saw was located and bigger pieces or timber were handled. The researcher had to find convenient spots to avoid interfering with Matti's various tasks. Also, the obvious noise of machines affected the recordings and the conversations. Safety measures had to be adhered to and special equipment was required for some of the tasks, such as sanding or cutting wood. Matti provided safety goggles and breathing masks for the dust component. The duration of each session ranged between two and four hours. At the conclusion of each session Matti explained the next steps, so that the researcher would be familiar with the content of the following meeting in order to avoid any possible surprises (see McDonald (2005) – *"Never go in cold").* The researcher also had to become familiar with the basics of the machinery, tools and materials in use. These circumstances created a further discussion to clarify and find common understanding between the researcher and Matti. Almost all the encounters with Matti were videotaped and pictures and notes were taken. We also collected physical data, such as Matti's sketches and drawings. In this way we could ask Matti to recall the situation, to confirm what was happening or to provide further explanations, or to renegotiate the meaning of the story (Creswell, 2012). Considering the fact that Matti and the researcher were constantly alone at the workshop we resorted to obtaining extra information during the fieldwork by asking Matti about the procedures and happenings. This avoided unnecessary awkward silences. Basically, it was a matter of trust. Sometimes, Matti stopped the work to think and the researcher's questions were the only way to access a verbalization about the situation. (Vásquez, Brummans & Groleau, 2012). According to Mc Donald (2005, p. 457) it is a straightforward strategy to obtain access to both the task and the reasoning behind it.

Factors such as time, inconveniences or unexpected turns in the process forced the researcher to make constant decisions that affected the shadowing process. The shadowing process lasted over 17 months while the first prototype was being built. Matti was interviewed twice in addition to the multiple conversations in the workshop. There were three occasions when we were not able to meet Matti at the workshop. In all three occasions, Matti voluntarily documented what he had done by taking pictures. Before the next session commenced, Matti explained the tasks, showed the pictures he had taken and discussed his feelings and impressions. This was a clear sign of the strength of the relationship established between him and the researcher.

The documentation process is still ongoing, since Matti agreed to further share the development and improvements applied to the model, following the first 1:1 prototype. By June 2019, the 18th Pekki table was being produced and some features of the first prototype had been modified.

Data Analysis

First, we were slightly overwhelmed by the amount and the characteristics of the collected data. There were video tapes, field texts, pictures and sketches all illustrating a lineal process in terms of time. We summarized, sorted out and listed all the actions that had taken place and were videotaped in each session. The sessions had been video recorded intermittently, since the action took place in different rooms of the workshop and there were different machines in use, located in different places of the rooms. The researcher moved the camera and focused it on different elements as well as Matti's various actions. A total of ten sessions were concluded, each of them between half an hour to five hours in duration. There were also several non-recorded conversations and hundreds of photographs. The videos were archived, and the pictures renamed when possible with a descriptive word based on what was captured. Some of the pictures were shown to Matti in order for him to recall the situation and to reaffirm the understanding of what was going on. The details in field notes had to conform to Matti's *a posteriori* explanations as well as the video recordings. Thus, in a separate conversation with Matti, a list of all the steps to build the prototype was made and examined. The researcher visited the workshop several times to confirm some of the details from the notes, such as names of tools and uses of jigs and machines. Subsequently and in order to answer research question 1, a preliminary outline of the whole process of constructing the prototype was made based on actions and tasks, where all the steps were placed in chronological order. We organized the photographs based on the steps of the process they illustrated with the help of the field notes and the videos. This was done in order to provide context for the photographs in the broader scheme of the process. This connected us with the second research question, which is addressed based on the help of the photographs and field notes. We were able to determine a series of instances when Matti encountered a problem solving situation. Those were also listed in the outline. This provided a clear idea of the course of action of the process. Thereafter and for the purpose of analysis, the outline was reproduced on a (large) poster and the photographs were printed and placed in chronological order. The idea was to encapsulate everything in a poster and capture the entire process. We color coded the different phases of the process and we labeled the problem solving situations. After that we were able to start our narrative about the creation of the Pekki Table.

Findings & discussion

The pekki table story

The idea to design a unique item

The Pekki table story answers our first research question and begins with Matti getting the idea to create an item with a unique design with a personal meaning attached to it for each customer (Buchanan & Margolin, 1995; Lubart, 2001; Risatti, 2007). He considered those as unavoidable conditions since the cost of a handmade product is usually high compared to the industrial ones. Matti explained that the engine of this project was the fact that most of the timber produced in people's yards and lands are mostly used as burning wood. When these trees must be felled for some reason, Matti thought that by transforming the timber into something new and useful, instead of burning wood was a good alternative for the customer. Amabile (1983) referred to this phase as problem identification phase and for Wallas this was the preparation stage (1926). Trees stand for decades in the same spot on a land and often, they become landmarks attributed to memories of past events or generations. Matti considered a piece of furniture and his mind started to wander around the idea of a table, where a family spends time and gathers around to eat dinner. He had experience building kitchens and realized that kitchens had too many intrusions - as he called them - he would have to cooperate with other professionals and make too many compromises. In addition to this, his prerequisite was for the item to be ecological, and a kitchen was not. At this moment, the idea of designing a table was born. However, Matti did not have a clear vision in terms of the look and design of the table. This was the prologue of the story. Matti acknowledged the "want to create something taking into consideration the different conditions and factors" which Jonassen (2000) would refer to as the integration of different content domains. This 'something' became a table. It was also the first time we got to know about this project, and it was then that Matti was asked to share his thoughts for documentation purposes. Matti accepted.

Thinking about "how is the table going to be"

Matti thought about several ways of what he wanted the table to look like. He drew sketches (e.g. Lawson, 1997; Goel, 1995; Aspelund & Kontzias, 2006; Cross, 2011) and made some approximate calculations of the possible measures of the table. However, he was not convinced. He had explained his ideas to other colleagues and friends, exchanged some views and was left with his thoughts. Time passed by pondering over the lines and shapes of each of the pieces, including both conscious, unconscious thinking and intuition, which made it vague to follow and difficult for Matti to share (Jonassen, 2000). This part could be regarded as the incubation stage of Wallas (1926) and the preparation phase for Amabile (1983). However, even if most of the explaining was done subsequent to the cognitive work, the embodiment via sketches and drawings was present. This made this period fuzzy and vague in terms of documentation (Guilford, 1950) as Matti discussed part of his thoughts and showed some sketches; but the certainty of all details remained hidden inside his head. There is no way to follow someone else's own private thoughts and divagations over a topic, let alone over a not yet formed idea for a long period of time. Most of the sharing was done *a posteriori*, when Matti's divagations were over, and when he was able to verbalize his thoughts. Matti kept drawing and sketching which helped him clear up his thoughts and ideas. This part of the story was regarded as the search for the right-fitted image of the dream product, the idea generation of the desired artefact (e.g. Dorta et al., 2008; Lahti et al. 2016; Goldschmidt, 2017). Matti claimed that he was not satisfied with just a mere idea of what he wanted (the final product) but he needed to feel that *"that is the one, the one that feels right"*. This part of the story was least accessible for the researchers. Problem solving processes per se were not detected. However, Matti used progress elements that are typically regarded as problem solving strategies; such as defining the limit of what he was looking for, considering the pros and cons of the product features, drawing pictures, trial and error, working it backwards, generation of solutions and use of objects to simulate. Some of these elements are also present as tools in the design process (see e.g. Aspelund & Kontzias, 2006; Dorta et al., 2007; Pei et al., 2010;)

Envisioning the table

Frustration with the process emerged for the first time after some months. Matti claimed that he encountered the need to have a break and rest from the continuous search for the right idea (see Sapp, 1992). He was fatigued due to his everyday job and the normal stress of everyday life, and needed a holiday, therefore, a family trip to Japan ensued. Detached from his daily routines and everyday tasks, he suddenly had a clear idea of what he wanted, and it felt good. He envisioned the table, its main features and lines. He reported as soon as he came back from Japan that he *"knew what the table would look like, the lines, the features"*. He said that he could see it. Wallas (1926) regarded this period as the illumination stage and Amabile (1983) labeled it as the response generation. We consider this the turning point in our documentation process. This stage opened the door to the 'real' game: the envisioned table started as a visualization and now Matti could sketch it in more detail and share it with the researcher. This period was free from problem solving, as if the greatness of the finding erased considerations over possible issues with the execution. These possible problems were to appear in the stage that followed.

Constructing the first prototype

Constructing the first prototype was not apparent or swift. It meant the embodiment and execution of what was just a visualization, an idea combined with a few sketches. This was the time where his abstract idea became concrete and was materialized, which Amabile (1983) regarded as response validation stage and Wallas identified as verification stage (1926). The table - now named as Pekki - was literally being created. For Matti, what had been a vision complemented and developed in the form of sketches and drawings was going to be un-built piece by piece, and reconstructed in wood. Matti first did a 1:10 model to materialize the idea. It resembled a sauna bench rather than a table, but Matti found the lines and proportions pleasant. This scale model was made out of solid wood, without dovetail joints and, instead, with domino oval pieces. He was satisfied with it, but he was not sure of how the legs and joints would be in a 1:1 scale model. The materialization helped the development of the process (Lahti et al., 2016) and afterwards, Matti had to fabricate a scaled 1:1 prototype of the envisioned table to see if the proportions and look of his idea were right. We documented this process in real time.

Initially building a prototype seemed as an extra step. The 'glory' of the Pekki table remained in the enlightenment where the idea appeared. However, the relevance of the prototype construction was massive, as it translated into an evaluation of the feasibility of the outcome (Howard et al., 2008; Dorta et al., 2008; Cross, 2011).

Photo I. Matti preparing timber to be used for a Pekki table.

Matti used regular timber bought from a nearby wood supplier for building the first prototype. He explained that for a real unit, the timber should come from a tree with some sort of connection to the customer. For the prototype, he needed to see the behavior of the wood, and therefore the timber used had to be reasonably easy to manage and strong enough to tender a good quality result, as a prototype would be the sample unit for further business opportunities (Temeltas, 2017; Risatti, 2007). Matti prepared the timber, soothed and squared edges for the top of the table (Photo I). He trimmed the sides of the log, cut it in half and diagonally split it. He had to come up with three different jigs to assist with the job (A, B and C, see Table 1). Matti needed the jigs in order to accurately use the band saw (see Paavola & Ilonen, 1981; Saló i Nevado & Pehkonen, 2018). He measured the diagonals of the planks and cut exact pieces. Then he measured the width and planed the wood. After that, he needed another jig (D) to measure the thickness and plane the timber again. The last operation was to measure the length and cut the end with the circular saw. At this point, Matti ended with four timber pieces to make the top of the table ready for the next adjustments. Matti made modifications and amendments to the top of the table planks and glued them two by two. The same jigs (A, B, and C) were used for slanting the ending of the glued pieces. He proceeded to put together all four boards to form the final top of the table using one more jig (H). Once the top was glued together, he made the hole for the dovetail joints of the legs (Photo II). He milled the timber and routed a 7-degree angle socket. This operation was the most demanding one in terms of jigs. He constructed one more jig for that purpose (I).

Photo II: a router and jig I to make a 7-degree angle socket for the dovetail joints.

Photo III: the top planks glued together

At the end of this phase, he sanded the top of the table. Matti repeated the same operations as with the table top planks but with different measurements. He trimmed the sides of the log, cut it in half and diagonally split it. He used the same jigs as the table top but with a different angle (A, B, C and D). He pitched the legs with the help of another jig (J) and cut the dovetails for each of the joints of the legs with the aid of jig K. Before polishing the legs, Matti cut the endings to stabilize the table when in standing position. At that point Matti had all the different parts of the table ready i.e. the table top and the legs. He assigned a number to the table (Pekki 0) and marked it with a burner inside one of the dovetail holes of the table top. However, when Matti assembled the table, the legs felt too heavy. He was disappointed and claimed that they looked like massive *"elephant legs"*. He then started to plan and make holes with chisels on the legs to try to lighten the appearance. He did a lot of sanding and soon the legs started to look like *"an animal bone"*. He took the router and made the outside edgy curves round with a quarter round blade. Now Matti was pleased. The prototype, which had helped Matti to make the final refinements (Cross, 2011), was ready. The *Pekki table* was born.

The name Pekki came from a stream that passes beside Matti's workshop. A stream is called *puro* in Finnish and *bäck* in Swedish. In the olden times, Finns called it *Pekki,* shaping the Swedish word into a more Finnish-like sound. Matti liked the named since it reminded him of something sturdy, made of wood, like a bench (*penkki* in Finnish language).

Prototype versus Pekki table final model

In line with several studies related to the design process (see Aspelund & Kontzias, 2006; Dorta et al., 2008; Cross, 2011), the final model of Pekki Table (Photo IV) was developed based on the prototype. There were three substantial differences. First, in the prototype the bottom of the table top was slanted from the ends towards the center, but not in the final models of the Pekki table. Secondly, the top planks had a four-millimeter space between them in the final model (see Photo III). He drilled holes for the dowel joints that later would connect the planks. He cut the dowels in octagonal shape, sanded the endings and glued the dowels to one of the sides of the planks. These operations needed three different extra jigs (E, F and G). In the prototype model the top planks were glued together. The third difference was the round shapes. In the prototype all the edges were rounded as opposed to the final model were all the edges are sharp.

Photo IV: The final model of Pekki table, photograph by Jonna Öhrnberg.

Problem solving, design and creative processes

In terms of answering our research question 2, throughout the process of building the prototype, Matti did numerous routine jobs such as cutting, sanding, measuring, gluing, etc. Simultaneously, he encountered several problems when intending to carry out those tasks because of the measurements or the slant of the timber. It became obvious that some situations interfered with the pace of the work and most of these drawbacks corresponded to moments when Matti had to create jigs (Saló i Nevado & Pehkonen, 2018). Although problem solving and creative processes share many similarities, we do not consider creating jigs as a creative process. We consider jigs in the first place as problem solving emerging from the professional practice (Llorente, 1996). Jigs are meant to serve definite purposes, and their value is in feasibility and practicality, not in novelty or exterior design (see Saló i Nevado & Pehkonen, 2018). Problem solving is not a uniform activity nor were the jigs Matti used, but each of them conditioned the existence and possibility of the Pekki table (Jonassen, 2000). The eight jigs that Matti used for the prototype construction are presented in the first column of Table 1. Jigs E, F and G are not included, since they were not used in the prototype construction.

All the jigs used by Matti had a common denominator: time. The difficulty level of the jigs' directly influenced how much time Matti used to find a solution. The first jigs (A, B, C and D) were easy for Matti as an experienced cabinetmaker, whereas some jigs (for example I and K) were more complicated and demanded more time to construct. Jigs A, B, C and D are examples of what Jonassen refers to as ill-structured problems that had become well-structured with practice (2000, p. 67). The mathematics embedded in jigs I and K were related to angles of inclination and trigonometry (Photo II). Matti did not use trigonometry to solve and find solutions for the jigs. At some point he mentioned that most likely, using trigonometry to calculate the angle he was looking for would have been easier and faster, if the mathematical procedures were ready in mind. However, the materialization of the mathematical knowledge was the part that became the most time consuming. Hence, he discovered by trial and error that each side of the wedged dovetail needed to be increased by 4-degrees to enable the tightening of the elements (mortice and dovetail).

Success in building each of the eight jigs' was translated to the process of the Pekki table construction, in the form of progress. Each of the jigs worked as a key to the next step in the process, like a door. In the Pekki table prototype construction, the jigs order, in terms of the production of each of the pieces, mattered. However, some of the jigs had to be used more than once and that meant that there was no need to build a new one, but to apply the existent ones and modify the adjustments to obtain specific measurements. This is the case of jigs A, B, C and D, which had to be used for both cutting the timber for the top of the table and for the legs.

Seven jigs out of eight had a clear influence on the precision of certain stages of work. The three first jigs (A, B, C) were made to assist the band saw and to allow precision to the cuts made to obtain the exact length, width and slant in the planks. Jig D was used to assist the thicknesser planer in obtaining the right thickness of the wood. Matti did not consider these jigs as complicated. Jonassen (2000, p. 69) would call them routine problems since they were familiar to Matti as an experienced cabinetmaker. Matti only needed to care about the exact measurements and positioning of the timber to obtain the right cuts. Also Jig H, exerted the right pressure and maintained the timber in the right position to be compressed and glued correctly. These first 5 jigs required mathematical precision to be a success and, therefore, they were technical and problem situated, falling into the category of diagnosis-solution problems (Jonassen, 2000, p. 75). There was no room for considering whether these jigs looked good or their design was groundbreaking. Their influence on the process of creation of the Pekki table was related to precision and since those timber pieces were the base of the project, a perfect functionality of the jig was required. The fact that these problem solving situations were successful, does not automatically regard their novelty as a creative trait. This is in contrast to Wimmer's statement that "successful problem solving can be regarded as creative process" (2016, p. 3). Jigs I and K were meant to assist the router to making a socket with a certain incline for the mortice and the dovetail (Photo II). Both jigs required a more advanced level of mathematical knowledge and Matti did not have any previous expertise (see Raami, 2015). The mortice and the dovetail needed a 7-degree angle to match each other and the shape of the mortice was complex since it was a truncated and rounded wedge with a leading edge opening to the trailing base of 4-degrees per side from the truncated corners.

To answer question 2, the problem solving situations emerged during the creation of the Pekki table influenced the process in terms of time, precision and progress. The problem solving situations created drawbacks, were time consuming and interfered with the pace of work. The perfect functionality of the jigs allowed precision in the cuts and the different parts of the table, and success translated into progress within the Pekki table process.

The intertwiness of processes

In order to understand the interconnectedness between processes and to answer the third research question, we took each problem solving situation (jigs) under consideration. We looked at what happened in each of them in terms of design and creative processes and also at why and for what purposes the jig was needed. In fact, we noticed that all problem solving situations consisted of a set of three smaller problems. Firstly, how to proceed (how to saw, how to cut, how to glue…); secondly, what jig is needed to proceed within the construction process and thirdly, how the jig is constructed. Jigs, as tools, are the solutions. Table 1 describes the jigs and their connections to the creative process, the design process and the construction of the prototype process.

Table 1. The problems solving situations and their influence on the creative process, the design process and the construction of the prototype process.

PROBLEM SOLVING SITUATIONS (PSS) = JIGS	CREATIVE PROCESS	DESIGN PROCESS	CONSTRUCTION PROCESS
JIG A: assisting jig for trimming timber ends.	No special influence (NSI) in terms of creative process.	NSI	The cuts are needed in any case for any type of table. Needed for the measurement precision.
JIG B: assisting jig cutting the timber in half to get two planks.	NSI	NSI	Relevant for proceeding with the construction of the table. Needed for the measurement precision.
JIG C: assisting jig in diagonally splitting the planks for the top of the table.	Mattis's mental idea about the visual lines of the table top.	First jig with clear influence on the design process. This jig allowed Matti to confer to the top of the table an original trait. The inclination of the bottom of the table top. Without the jig, the cut could only be perpendicular to the base instead of diagonal. The diagonal conferred the desired visual effect. Distinctive design.	Relevant for proceeding with the construction of the table.
JIG D: assisting jig for holding the timber when using the thicknesser planer for measuring and planning the precise plank thickness.	NSI	NSI	Relevant for proceeding with the construction of the table. Needed for the thickness precision.
JIG H: assembling jig for holding the top planks together until the glue is dry. (Photo III)	NSI	NSI	Relevant for proceeding with the construction of the table.
JIG I: guiding jig for the router to obtain the exact desired mortice shape on the bottom surface of the top of the table routing a 7-degre angle socket for the dovetail joints with a distinctive precise shape: truncated and rounded wedge shaped with a leading edge opening to the trailing base of 4-degrees per side. (Photo II)	Matti's mental, innovative idea about the tightening system of the table and the joints of the legs.	Influence on the design. This jig allowed Matti to obtain the precise mortice shape. The shape that the jig permits the router to make will let the legs to be connected without joints, nails or glue by exercising friction and tightening the leg to the table top. This is one of the innovative design traits of the Pekki table.	Relevant for proceeding with the construction of the table.

JIG J: subjecting and guiding jig for the manual leg pitching. The jig secured the timber to be able to use planers and sanding paper to give the right slant to the legs of the table.	Matti's mental idea of the visual lines of the legs.	Influence on the design process as this jig permits the visual effects that Matti wanted to obtain on the legs of the table. This jig conceded the handwork of the cabinetmaker on the timber piece.	Relevant for proceeding with the construction of the table.
JIG K: guiding jig for the router to make a dovetail in each leg. Complementary jig to jig I. The jig guides the router to obtain the exact desired dovetail shape at the end of each of the legs.	Matti's mental idea about the tightening system of the table and the joints of the legs	Influence on the design. This jig allowed Matti to obtain the precise dovetail shape. The shape that the jig permits the router to make will let the legs mortice to be connected without screws, nails or glue by exercising friction and tightening the legs and the table top. This is an innovative design trait of the Pekki table. The legs are detachable.	Relevant for proceeding with the construction of the table.

As shown in Table 1 four of the eight jigs that Matti used had no influence on the design process of the Pekki Table. Those jigs (A, B, D and H) assisted the stages of the job that were needed for the construction of any type of table and had no innovative component and therefore routine design (Howard et al., 2008). However, as shown above, each of them contributed to the construction process through the precision of the item and its features, the progress of the processes, and on time consumed. The four remaining jigs had a clear influence on the design processes of the Pekki table (shaded cells in Table 1).

Matti's endeavor was to define the traits and lines of the table so they would become different, attractive and functional and therefore a new design (Risatti, 2007). The creativity of the process of the Pekki table design was directly connected to those new traits, and hence, the moments when those traits were made possible in the design process, they were meaningful (Risatti, 2007; Temeltas, 2017). The three design traits that defined the Pekki table as a new design, were the slanted bottom surface of the top of the table, the absence of nails, screws or glue in the joints (which allowed the table to have detachable legs), and the tightening system facilitated by friction (not just stuck on and attached by force). These made the Pekki table portable and possible to assemble at any given time. With the help of jig C, Matti was able to split the planks diagonally – as he wanted – for the top. Jigs I, J and K solved the problem allowing for a new design, thus making the design innovative. In particular, the friction exercised by the correct angle of inclination of the inner wall of the mortice and the dovetail made the tightening strong and firm. So, the design traits Matti had dreamed of was made possible.

Our analysis shows that problem solving processes are mediating processes between creative and design processes. Referring to the model of Wallas (1926), the design process can be seen to take its position between illumination and verification phases of the creative process. In the design process, Matti's illuminated, ideated and partly visualized ideals were verified and realized. Problem solving assisted in gradually making it possible.

Concluding remarks

A creative process ends in a new idea, something novel and original. In the case of the Pekki table, at the beginning of the process, Matti wanted to create "something new", innovative. Therefore, the process was regarded as a creative process, which started when Matti realized that he wanted to create something. This was a problem solving situation in itself as there was a gap between the moment Matti knew that he wanted to "create" something and when the "created object materialized". Wimmer (2016) regards a problem situation as a necessary condition for a creative process to occur. In our case problem solving and creative processes are parallel to each other. This is what we have argued previously (Saló i Nevado & Pehkonen, 2018). At the same time, the instant when Matti could visualize and decide that his creation was going to be a table, the creative process became parallel to a new design process, since a table is an artefact from a category that already exists.

Our data indicated that the creativity of the problem solving solution, i.e. the level of creativity of the jigs, was not relevant at all for Matti. Instead, the success of the outcome of the problem solving situation (i.e. the jigs), had influence on the creative traits of the table and direct impact on the design. Matti's jigs could be considered by definition as open-ended problems, or ill-structured problems as they were open to many different solutions (Becker & Shimada, 1997; Jonassen, 2000), and they mediated between the process of creation and the design of a table.

Our findings are based on very specific data in a specific woodworking context. Further research must be pursued in different vocational fields to obtain a deeper understanding about the connections between problem solving at work, design and creativity.

Without Matti's commitment and enthusiasm, writing this paper would not have been possible. In this study, Matti was not a mere informant or subject under observation. Rather, he was our active partner, who dedicated himself to our project in a unique way. He not only shared the entire Pekki Table creation process and his thoughts with us, but also revised and commented on our text. Matti checked and verified that what he had thought and done during the process was noted and understood correctly. In terms of validity this has been extremely valuable.

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